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BETWEEN MIND AND MATHEMATICS. DIFFERENT KINDS OF COMPUTATIONAL REPRESENTATIONS OF MUSIC

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RÉSUMÉ – Entre l'esprit et les mathématiques. Sur différents types de représentations computationnelles de la musique
Dans cet article nous analysons différents types de représentations de la musique, aussi bien d'un point de vue cognitif que computationnel. Si les représentations mentales de la musique sont l'objet de l'esprit musical, et donc par définition si elles constituent une question philosophique et cognitive, on peut faire l'hypothèse que les représentations mathématiques aussi aient des corrélats cognitifs permettant la compréhension de la musique non-tonale. Parmi les nombreuses typologies de représentations mathématiques de la musique, nous analyserons en détail quelques exemples relevant du paradigme transformationnel, un sous-domaine formalisé de la musicologie computationnelle provenant de la tradition ensembliste américaine. La démarche transformationnelle en musique ouvre aussi de nouvelles questions sur les ramifications cognitives et philosophiques des approches algébriques et catégorielles en théorie de la musique, analyse et composition.

MOTS CLÉS – Analyse transformationnelle, Espaces conceptuels, Musique, Objet sonore, Représentations mentales, Structuralisme phénoménologique, Théorie générative de la musique tonale

ABSTRACT – *In this article we analyse different types of representations of music, both from a cognitive and a computational point of view. Whereas mental representations of music are the objects of the musical mind, and are therefore by definition a subject of cognitive psychology and philosophy, it can be argued that mathematical representations of music also have some cognitive correlates enabling the understanding of non-tonal music. Amongst the many typologies of mathematical representations of music, we will analyse in detail some examples belonging to the transformational paradigm, which is a formalized subfield of computational musicology coming from the American set-theoretical tradition. Transformational music analysis also raises new questions about the cognitive and philosophical ramifications of algebraic and category-theory approaches in music theory, analysis and composition.*

KEYWORDS – Conceptual spaces, Generative theory of tonal music, Mental representations, Music, Phenomenological structuralism, Sound object, Transformational analysis

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1. INTRODUCTION

According to Nussbaum [2007], modern western music is without any reasonable doubt a question of representations: “Pieces of music are not physical objects, and so have no single 'ground' manifestation. The question of how to represent a piece of music for processing by computer therefore does not have a single answer” [Marsden, 2005]. So, different kinds of representations are possible and probably desirable for different theoretical or practical aims. Despite of the relative lack of communication between cognitive and computational musicology and the mathematical theory of music, our aim is to get these disciplines to communicate together. Our perspective is pluralistic: the representation of music is said in many ways, and according to neural darwinism the representations in the brain are in competition. We think that mind/brain has the possibility to represent music at many levels and in different forms, as musical structure and musical perception are not necessarily congruent: mathematical structures of music could be perfectly objective but non entirely (or not at all) perceivable, as it is the case for Z-relations, and Tn and TnI relations [Forte 1973; Rahn 1980; Morris 1988]. The objectivity of musical structures and their cognitive relevance has to be approached through a plurality of conceptual instruments.

2. MENTAL REPRESENTATIONS IN COGNITIVE MUSICOLOGY

In cognitive musicology the use of the concept of mental representation is quite widespread but ambiguous and vague. The mainstream cognitive musicology maintains that mental representations of music are non-conceptual or are, in Dretske's [1995] taxonomy, sensory mental states (see Figure 1):

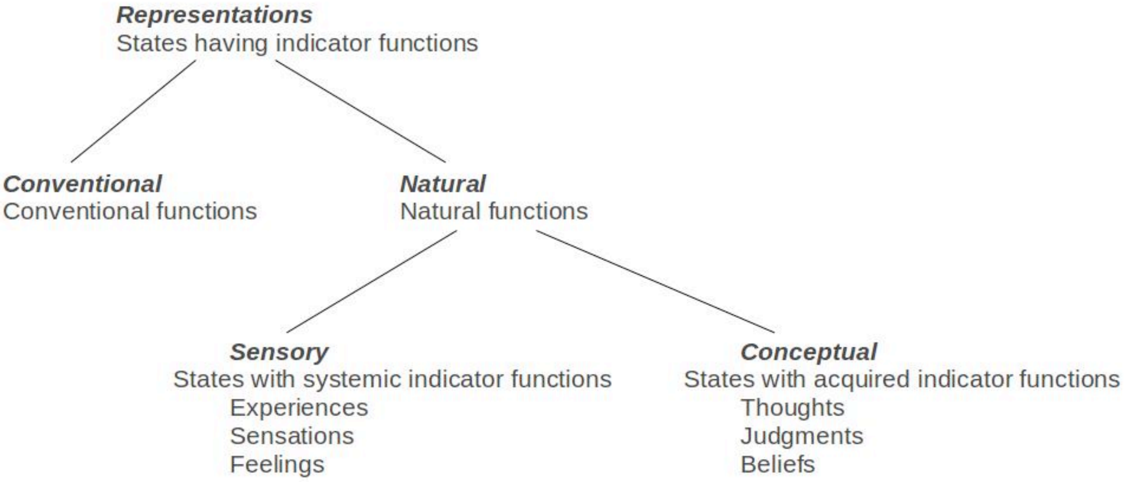

FIGURE 1. Dretske’s [1995] taxonomy of representations

In Lerdahl and Jackendoff’s Generative Theory of Tonal Music (GTTM in short) – one of the most popular cognitive theories of music – the term 'representations' is used without particular specification about their nature (syntactic and not semantic, with non-conceptual content): “We have restricted ourselves to a formal characterization of the listener's intuitions about musical structure (that is, of his mental representation of music)” [Lerdahl & Jackendoff, 1983, p. 332].

In many cognitive theories of music, mental representations of music are considered to be implicitly construed by the mind according to the perception of musical flow, as tacit – or implicit – knowledge. By contrast, in GTTM the mental representations of music are considered in the framework of a final-state theory, and the authors are not committed to explain musical cognition as a real-time process: "Our approach to music theory has not been concerned with questions of memory capacity, real-time processing, brain localization, and the like" [Lerdahl & Jackendoff, 1983, p. 42]. The symbolic representations of GTTM are alleged to represent the static mental representation of a piece of music (that is a kind of natural and spontaneous musical analysis made by the mind). So the standard musical notation *plus* the binary branching trees notation (Figure 2) would represent a graphic analogon of mental representations of music.

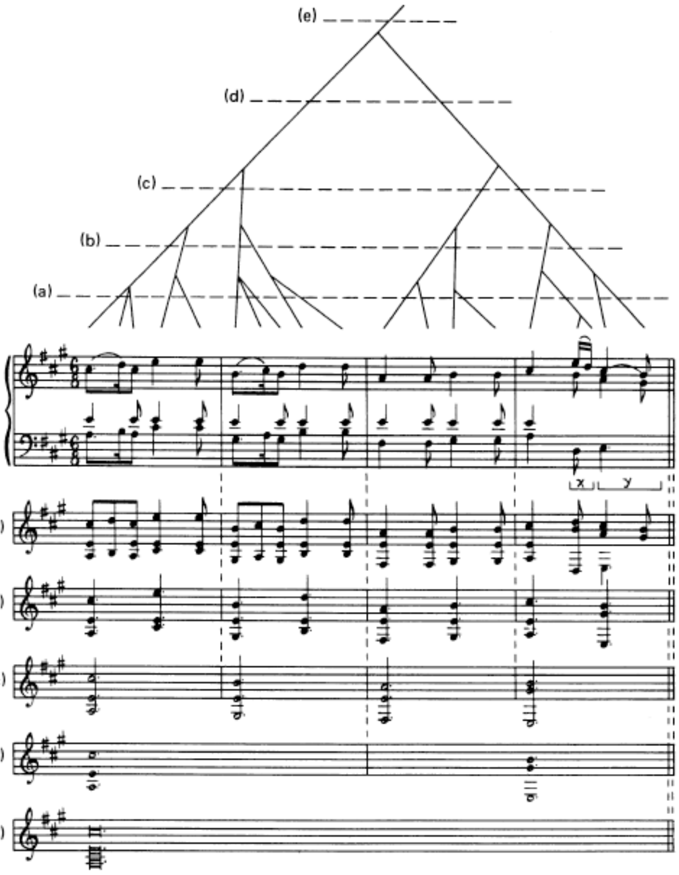

FIGURE 2. Time-span reduction of the opening of Mozart, K. 331, as an example of the representational structures of GTTM [from Jackendoff 1987] [Reprinted with the kind permission of MIT Press]

Lerdahl [2001] expanded the theory (allegedly) eliminating all qualitative points, thus obtaining a first series of algorithms for tonal tension/attraction in the pitch space, which have been also experimentally tested [Lerdahl & Krumhansl, 2007], and which resembles in some aspects Huron's theory of the mental representation of musical expectation [Huron, 2006]. Of course, GTTM has to assume that symbolic representation of music have a content analogous to that of mental representation of music. But according to DeBellis, GTTM "representations" of music are not true representations insofar they do not have – as true representations have – truth conditions [DeBellis 1995, p. 21].

3. ON JACKENDOFF'S NOTION OF "MENTAL REPRESENTATIONS"

DeBellis critical point of view falls in the philosophical side of the opposition that Jackendoff [1992] calls (evoking a Kuhnian "paradigm split") the Philosophical Version of the theory of mind: "What is the relationship of the mind to the world, such that we can have knowledge of reality, such that we can have beliefs and desires about things in the world, and such that our sentences can be true or false?" [Jackendoff, 1992, p. 158]. But according to Jackendoff the philosophical position does not fit the scientific standard of cognitive science, and thus he strongly prefers the Psychological Version of the philosophy of mind: "How does the brain function as a physical device, such that the world seems to us the way it does, and such that we can behave effectively in the world?" [Jackendoff, 1992, *ibidem*]. This position derives from the Chomskyan notion of I-concepts ("internal concepts") that allows science to inquire only on the already formed concepts and mental representations, without investigating their origin and referential connection with world.

DeBellis burdens on Jackendoff alone the weight of his mentalistic position, according to which the problem of the right reference of representations to the world is not a problem for semanticists but for general psychology. Indeed, Jackendoff makes musical objects a kind of mental-dependent objects [Bullot & Égré, 2010]: "the constancy and reality of a piece of music are purely mental" [Jackendoff, 1992, p. 165]. Jackendoff criticizes the very notion of "mental representation" as it is normally used implying intentionality: "I am trying, therefore, to take the notion of representation as an entirely nonintentional notion. A representation is not necessarily about anything; if you like, it does not strictly speaking represent anything. (Hence my hesitation in using the term except as a rigid designator for what cognitive scientists believe the mind has in it.) The point of this notion of representation is that it can in principle be instantiated in a purely combinatorial device like the brain as I understand it, without resort to any miraculous biological powers of intentionality such as Searle [1980] wishes to ascribe to the brain" [Jackendoff 1992, p. 160]. He proposes also a possible alternative term for indicating the relational character of "mental representations" in his views: "If one wishes to reserve the term *mental representation* for brain-states-with-intentionality, I have no objection to introducing a new term, say *mental distinctions*, for the nonintentional states that I am calling mental representations here" [Jackendoff 1992, footnote 2, p. 182].

For Jackendoff, mental representations of music have definitely no truth value: "It hardly makes sense to say that the representations one constructs in response to hearing a performance of the *Eroica* are true or false. Nor does it make sense to claim one has propositional attitudes toward musical representations, which are not, as far as I can tell, propositions" [Jackendoff 1992, p. 165]. Jackendoff's insistence on this point is even exaggerated, as he tries to use mental representations of music against philosophical positions *à la* Fodor which maintain that in mental representations what is essential is the semantic content: "the factors that make a piece of music cohere for a listener into something beyond a mere sequence of notes involve complex internal computations over abstract mental representations of the piece. Fodor's insistence on respecting semantic relations seems out of place here: "these abstract structures are part of mental life, but one would hardly want to make a metaphysical claim about there being something "real" in the world, propositional or otherwise, that they are representations of" [Jackendoff 1992, pp. 29-30]. Here the question became puzzling: we can easily allow that musical structures are not out there in the world; but what about sound objects at the basis of musical structures? It seems very implausible that sound objects are not *intentional object* [Bullot & Égré, 2010]. The position of Jackendoff is very close to that of Dennett [1987] who "describes this situation in terms of the brain being a *syntactic engine* that mimics a *semantic engine*: by virtue of its evolutionary history, it acts for the most part as though it is making genuine contact with properties of the physical world" [Jackendoff 1992, p. 182]. This account is perhaps critical in the case of the semantic properties of the language, but for the case of music it is quite acceptable, as the only direct match of the musical faculty with a peripheral mental module is plausibly that with the body representation (at work in the case of the dance[3]). So in Jackendoff's view, the notion of mental representations has to be considered as purely syntactic-combinatorial, and this account seems compatible with the recent and important reinterpretation of GTTM made by [Katz & Pesetsky, 2009], according to which music and language are structurally identical except for the nature of the compositional blocks of the two mental devices: concepts in the case of language, and sounds in the case of music.

4. REPRESENTATION OF THE EXPECTATIONS

The now well-known concept of "expectation" was introduced in musicology by Leonard B. Meyer [1956]: a sound event causes certain kinds of expectations of other sound events, and the satisfaction or the delusion of the expectancy is commonly considered as "a generator of musical affect" [Margulis, 2005, p. 663]. More recently, Huron [2006] sketches a complete theory of psychological expectations, starting from Meyer's seminal study. To expect something is to mentally represent something as coming, so expectation is a kind of mental representation: "These expectations can be satisfied or not; it is this that makes them representational" [Luntley, 2003, p. 414]. Of course, here the framework is sensibly different from that of the classical cognitive sciences, which postulate a "static" kind of mental representations or an online construction of representation of music: the way is here opened to the embodied musical cognition [Leman, 2008].

Huron [2006] discusses the evolutionary origin of sound representations, starting from the specific example of the localization of sound in which three levels of perception are involved: *unconscious* (interaural time, amplitude differences, spectral shape), *subconscious* (horizontal azimuth of sounds, elevation or perception of their vertical position, and perception of distance), *conscious* (place, speed, trajectory).

These kinds of mental representations of sounds are predictive of future events, so they implement the "*where-next* function", a useful function both for survival and for the musical understanding. So, for Huron, the representation of sounds has a biological function allowing different forms of representation, as it is easily understood in the canonical case of pitch representation: there are no *a priori* reasons for a listener hearing

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[3] "Searching for circumstantial leads, we observe that, among human activities, one that is closely related to music both in its structure and its affective response is dance. Dance is almost invariably performed to music, and its rhythmic characteristics parallel those of the music. Moreover, going beyond crude rhythmic correspondences, we have undeniable and detailed intuitions concerning whether the character of dance movements suit or fail to suit the music. Such intuitions are patently not the result of deliberate training, though they can be sharpened by training. This suggests that behind the control and appreciation of dance movements lies a cognitive structure that can be placed into a close correspondence with musical structure" [Jackendoff 1987, pp. 237-238].

a (western tonal) music to expect the next sound event as an absolute pitch, or a particular pitch-class, or a contour, or an interval, or a scale degree, or a particular member of a specific chord, etc. The possibilities are many, and which is the right one is for Huron a difficult empirical question. This is a difficult question because there are no known methods for discriminating in which format the human mind processes the sound information. It is plausible in fact that "a profusion of different representations might be useful for a listening brain" [Huron, 2006, p. 107], at least if we agree, as Huron does, with Gerald Edelman's "neural Darwinism" [Edelman, 1987]. From this perspective "a good mental representation would be one that captures or approximates some useful organizational property of an animal's actual environment" [Huron, 2006, p. 107]. The choice of the good format for a mental representation of sound is motivated from natural evolution, but how could the selection of the best mental representations work[4]? Here the concept of "expectation" is central, because a mental representation which is not adequate to the real world is more likely to cause wrong predictions.

Huron hypothesizes four general principles, or preference rules[5] for mental representation of pitch: (1) lower-order relationships, (2) neighboring over distant relationships (preferred for the same reason of 1), (3) lower-derivative states, (4) event-related binding [Huron, 2006, p. 122]. Those types of relationships and cognitive objects are preferred because they are simpler to compute. In each case, these principles would be oriented by the preference for simplicity over complexity, as in a consolidated tradition of the evolutionary inspired cognitive science [Chater & Vitanyi, 2003]. The principles are generally understandable as a preference for event (or object) more than for relations: it is not that mind/brain cannot represent relationships, but "It is easier to process, code, or manipulate representations when they are mentally attached to events or objects" [Huron, 2006, p. 124].

From a neurological point of view, the four principles are the more plausible in terms of neural networks, because they favor the computational simplicity. These principles are compatible with neural Darwinism because they imply that initial mental representations of sounds are simple, low-order and free from contextual information. Finally, the expectations reveal to be "neural circuits whose activation depends on the pattern of sensory inputs" [Huron, 2006, p. 127].

5. INTENTIONAL SOUND OBJECTS

The non-semantical content of a mental representation of musical sound is "a content that represents a sort of tension" [Luntley, 2003]: but is this content conceptual or non-conceptual? The non-intentional character of mental representations *à la* Jackendoff raises a lot of problems not only for the cognitive psychologist who does not want to cut off with the real world, but also for the philosophical account of sound events as *intentional objects*. According to Bullot and Égré [2010] the sound objects are clearly intentionally related to the real world: "Regardless of the exact status of its targets, auditory experience may be characterized as intentional, in the sense that it is about that which is heard, or that which determines whether our demonstrative judgments based on

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[4] Obviously it is not necessary to suppose that the selected representations are absolutely the best ones, but only the most adaptive in a particular context and at a particular moment: it is possible that other representational possibilities would abstractly be better than the ones selected, but for natural-historical reasons the evolution of our mind could have chosen the most contingently favorable and the least expensive solution, i.e., the most "relevant" in the sense of Sperber and Wilson [1986].

[5] The importance of preference rules for musical cognition is a central point in Lerdahl and Jackendoff [1983].

auditory experience are true or false (their truth conditions). In accordance with the terminology inherited from Phenomenology, the contents of auditory experience can therefore be described in terms of ‘intentional objects’, in the broad sense of that which the state is about” [Bullot & Égré, 2010, p. 7].

Against Jackendoff's statement that mental representations of music are non-intentional, the sound objects are defined as mental representations of the sounds in the world. If in the case of music the existence of higher level structures of the same kind as those postulated in GTTM is necessary to explaining musical cognition, in the case of non-musical sounds it seems possible to have a direct perception of the sound object. Huron's discussion of the complex cognitive process for localizing sounds in the space [Huron, 2006] is a good example of a “realistic” approach to the mental representations of sounds. It must be noted that the intentionality of the (mental representation of the) sound object does not need to be a conceptual intentionality, since, as observed by DeBellis [1995, 2005] and Luntey [2003], mental non-conceptual representations of music are very plausible: this kind of representations is very likely to cause behavior, but this behavior could not be rationally explained by the hearing subject.

6. GÄRDENFORS'S CONCEPTUAL SPACES

Ray Jackendoff sees in the cognitive science “the flavour of a paradigm split in the sense of Kuhn” [Jackendoff, 1992, p. 157], where two poles contend the fundamental framework: the “philosophical version” and the “psychological version”, although the distinction is an idealization of the divide. Here are some of the problems that were discussed from the philosophical point of view: the problem of the format of mental representations (symbolic *vs* connectionist); the syntax/semantics divergence; the question about the existence of non-conceptual contents; the disjunction problem. We have no room here to discussing all these problems but we can only hint some points.

Concerning the symbolic/connectionist divide, Gärdenfors [2000] says that “[i]t has been a common prejudice in cognitive science that the brain is either a Turing machine working with symbols or a connectionist system using neural networks”. According to Gärdenfors, neither symbolic nor associationist-connectionist approach can completely explain and model the mechanism of *concept acquisition*, that needs a formalization of similarity (recall Chomsky’s criticisms against the behavioristic notion of “similar stimuli” [Chomsky, 1967]). According to Gärdenfors, the solution is a third form of representing information, based on *geometrical structures*. Gärdenfors elaborated a theory of conceptual spaces in which concepts are considered as multidimensional domains with a multiplicity of quality dimensions. The conceptual level of cognition is the second level of Marr’s [1982] famous three levels of cognition, the level of “Representation and algorithm” between the symbolic (Computational) and the connectionist one (for Marr, that of Implementation). A conceptual space consists of a number of quality dimensions, where common examples of dimensions are “temperature, weight, brightness, pitch and the three ordinary spatial dimensions height, width and depth”.

But, if musical pitch were represented in a conceptual space, what about the relations between pitches and other musical dimensions that constitute musical relations and objects? Also the sub-musical (sound object) level is a matter of mental representation. Sound objects are intentional objects [Bullot & Egré, 2010], so they are mental representations of the sounds of the world: that is plausibly the evolutionary function of audition. Following Gärdenfors we hypothesize that the representation of the sound object should be at the subsymbolic level and the representation of musical object is at the symbolic level, that of Marr's "representations". Sound perception cannot be identified with music perception.

In order to explain the relation between sound object and musical object, we have to consider a particular notion of supervenience. The status of the "musical object" has to be of higher level than sound object. The musical object supervenes on (i.e., depends on/covaries with/is irreducible to) the sound object, and this supervenience should pass through the conceptual level of mental representations.

7. ALGEBRAIC FORMALIZATIONS AND GEOMETRICAL REPRESENTATIONS OF MUSIC

As shown in the previous sections, the notion of mental representations within the field of cognitive musicology remains ambiguous and vague. Nevertheless, as pointed out by Huron [2006], there is a plurality of representations of music proposed by music theorists during the twentieth century, even if many of these representations seem to be theoretical constructions which are quite artificial and without a great link with the experience of listening. Our aim is to try to detect a theoretical and experimental intersection for different kinds of representations of music, with different degree of mental reality and theoretical utility.

From the perspective of computational musicology, the concept of "representation" has deeply been connected with that of formalization, by often revealing different modalities of articulation between the two notions. If a mathematical representation of musical structure seems to anticipate the formalization process, in the case of algebraic methods applied to music, as it has been largely discussed in [Andreatta, 2003], one can see the representation act as crucially following the formalization process. As a paradigmatic example of this interplay, one can consider the two traditional geometric representations of musical structures, commonly used in the neo-Riemannian analytical tradition, i.e. the circular and the toroidal representations.

In order to understand the relationships between these two apparently very different representations, one simply has to notice that both directly derive from the formalization of a given equal tempered system with the algebraic structure of a finite (cyclic) group. The octave reduction enables first to reduce the combinatorial space of the division of the octave into *n* equal parts to the algebraic structure of cyclic groups of order *n* and, successively, to the toroidal representation via the Sylow decomposition of an Abelian group as a product of maximal *p*-groups, which directly offers an isomorphism between the circular representation and the toroidal space, commonly referred to as *Tonnetz* (see Figure 3).

The *Tonnetz* provides a geometrical conceptual space in the sense of Gärdenfors, enabling to represent musical processes as paths within this space. This opens very interesting questions concerning the possibility of automatically "generating" the musical space via new paradigms in computer-science (such as the spatial computing paradigm), according to the local logical dimension which is present in a given musical passage.[6] Interestingly, spatial computing can also open new perspectives in order to grasp the algebraic/geometric character of the so-called "transformational" paradigm in music analysis which generalizes at the same time the set-theoretical and the neo-

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[6] See Bigo *et al*. [2011]. For a different description of Gärdenfors's theory of conceptual spaces with respect to concept formations in Musical Creative Systems, see Forth *et al*. [2010].

Riemannian approaches. In fact, from an algebraic perspective, the traditional set-theoretical approach in music analysis is based on the hypothesis of using a given established catalogue of musical structures (chords, motifs, rhythms) as orbits under the action of one particular group, i.e. the dihedral group. But there are several possibilities to obtain different musically pertinent catalogues of orbits under the action of given finite groups (see Figure 4).

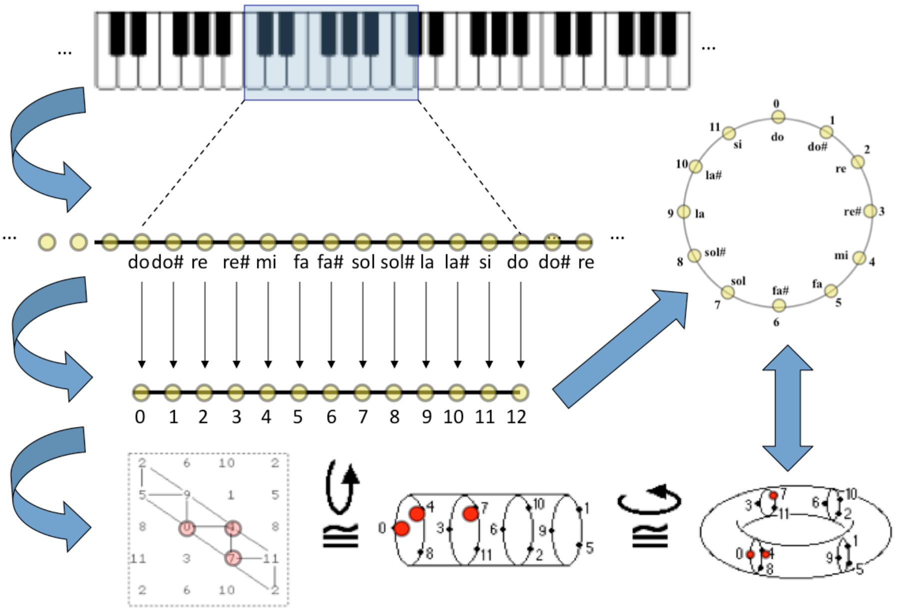

FIGURE 3. The interplay between the algebraic formalization and the geometric representation underlying the isomorphism between the cyclic group and the *Tonnetz*. Via the octave reduction, the combinatorial space of the piano is reduced to the algebraic structure of cyclic groups of order *n* and, successively, to the toroidal representation via the Sylow decomposition of Z/12Z as a product of Z/3Z et Z/4Z, which directly offers an isomorphism between the circular representation and the *Tonnetz,* which corresponds geometrically to a toroidal structure.

On the contrary, transformational analysis implies a double movement. On one side, one aims at *constructing* an abstract configuration of musical objects (called "transformational network"), and on the other side this formal architecture has to be *utilized* in order to help either the perception of the musical form or the interpretation of (a passage of a) given musical piece.[7] In other words, the interest of *constructing* a spatial network of musical structures lies on the possibility of *using* it for "structuring" the listening process or "guiding" the performance of the piece. The construction of a transformational network is based, in fact, on the implicit attempt by the analyst to make the underlying musical logic "intelligible" to the listener and/or to the performer. This logic has a geometric character, and for this reason, it seems crucial to us, in the case of a transformational analysis, to always couple the underlying geometrical representation with a computational model, which can been integrated in different environments for computer-aided music analysis. Before analyzing some new theoretical implications of this approach for a mathematically-oriented experimental psychology, as well as for a structural perspective on cognitive processes, we will show

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[7] This double movement is wonderfully captured by the title of David Lewin's essay providing one of the most interesting case study for a perceptual validation of the transformational paradigm ("Making and Using a Pcset Network for Stockhausen's *Klavierstück III*".) See Lewin [1993].

in more detail what a transformational analysis becomes whence it is approached from a computational perspective.

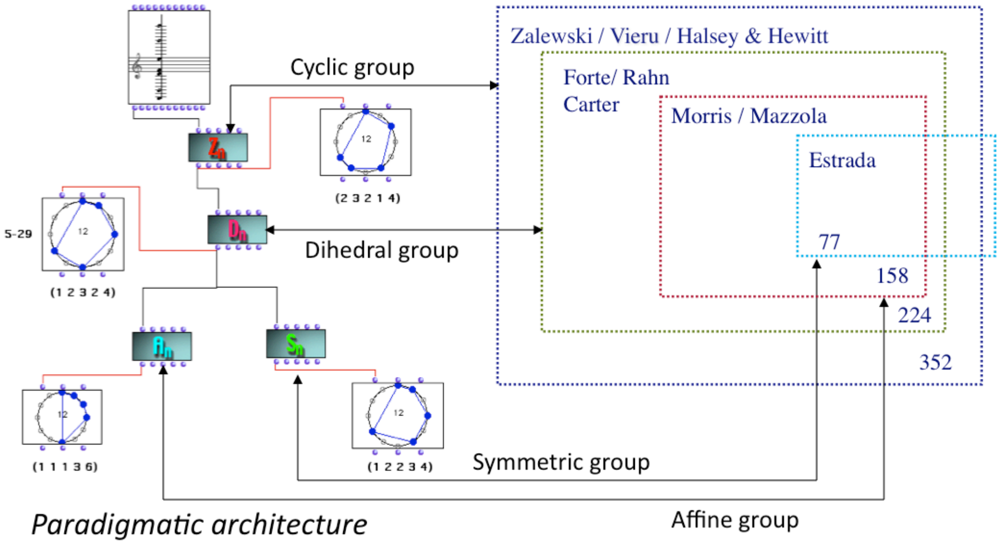

FIGURE 4. The paradigmatic architecture showing that the set-theoretical approach is but a special case of the action of a group (the dihedral group of order 24 generated by musical transpositions and inversions) on the collection of subsets of the equal tempered system.

This action enables to algebraically formalize Allen Forte's catalogue of pitch-class sets as described, for example, in Forte [1973]. The figure shows four other historically musically pertinent catalogues of musical structures, obtained as orbits under the action of given finite groups (cyclic, affine and symmetric groups). They correspond respectively to the catalogue of 352 transposition classes of chords (obtained, independently, by Maciej Zalewki, Anatol Vieru, George D. Halsey and Edwin Hewitt), the catalogue of 158 affine classes (obtained by Robert Morris and Guerino Mazzola) and, finally, the catalogue of 77 textures by the Mexican composer Julio Estrada. This architecture has been integrated into *OpenMusic*, a visual programming language used for computer-aided music theory, analysis and composition.

8. SOME COMPUTATIONAL ASPECTS OF TRANSFORMATIONAL ANALYSIS

In his celebrated analysis of Stockhausen's *Klavierstück III*, the American music-theorist David Lewin distinguished two radically different strategies revealing the logics underlying this short piece. Both approaches rely on the hypothesis that there exists a "generating" structure for the piece, more precisely a pentachord (i.e. a collection of five different pitches) which covers all the note-events contained in the score through two main musical transformations: transpositions (i.e. adding a given number of semitones to every element of the chord) and inversions (i.e. reversing the order of intervals contained in the chord). They geometrically correspond respectively to rotations of a polygon inscribed in the circle and to their reflections, according to a symmetry axis which keeps invariant the chromatic tetrachord included in the basic pentachord. The inversion is therefore always defined "contextually", i.e. as the symmetry fixing a given subset of the chord and moving the single note not belonging to the chromatic tetrachord.[8] According to the underlying algebraic structure provided

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[8] The reason for choosing this "contextual" inversion is primarily motivated by the perceptual relevance of this musical transformation, as Lewin observes. It would be interesting to discuss the "contextual" character of most of the algebraic tools developed within the transformational paradigm with respect to

by the dihedral group, transpositions and inversions can be combined by generating, in such a way, new symmetries (see Figure 5).

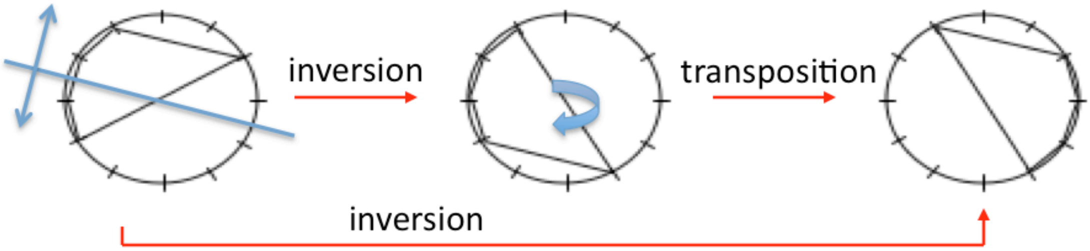

FIGURE 5. Inversions and transpositions as geometric operations within the circular representation.
A pentachord (i.e. chord containing five notes) is initially inverted with respect to a symmetry axis leaving invariant a subset (corresponding, musically speaking, to a chromatic tetrachord) and then transposed.

In the first approach, musical transformations, once algebraically formalized and geometrically represented, are organized in an order which reflects the temporal unfolding of the piece. This "chronological" vision of the pentachords organization is called "transformational progression" but it is not the only possibility of associating a geometric conceptual space to a given musical piece. In a second, and more abstract, approach, transformations build a relational network within which it is possible to define a variety of different paths. In this case the linear notion of transformational progression is supplanted by the spatial concept of "transformational network". A transformational analysis can be conceived as a dialectical process between transformational progressions and transformational networks, without imposing any predominance to one of the two analytic strategies. The interest is, on the contrary, to assign to these two "ontologically" different strategies different weights within the analytical activity. Figure 6 shows the first measures of the piece with the beginning of the segmentation process, as proposed by Lewin. Note that this segmentation is very different, in nature, from all the segmentations proposed by Lerdahl and Jackendoff in their GTTM, where segments never have elements in common. Lewin's segmentation by "imbrication", on the contrary, explicitly lies on the hypothesis that, from an analytical and perceptual point of view, segments without common notes are less interesting than regions sharing one or more elements. This simple fact clearly shows the spatial character of the transformational approach, compared to the "sequential" nature of the analytical engine of GTTM.

The segmentation is obtained by considering different instances, or transformations, of the initial pentachord. All these transformations do not affect the "set-theoretical" nature of the base musical structure, since all the forms are equivalent (up to a transposition or an inversion). Because of the fact that the segmentation directly follows the temporal organization of the musical piece, we are clearly dealing with a "transformational progression".

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similar notions in cognitive science, particularly in Dreyfus's phenomenology-oriented critique of AI [Dreyfus & Hall, 1982]. See Andler [2000; 2006] for a detailed analysis of the relationships between Dreyfus's position with respect to phenomenology, cognitive sciences and the problem of context. For a recent account of Dreyfus's influence on Lewin's transformational constructions, see Kane [2011], Andreatta and Petitot [2012].

A different strategy consists of considering the transformations as possible ways of structuring the abstract space of pentachordal forms. The "transformational progression" becomes a specific path within this more abstract relational space. The collection of formal relationships between different instances of the base pentachord constitutes a space of potentialities within which the piece moves. One of the main differences, with respect to the transformational progression, is that the organization of pentachordal forms within a network has no direct link with their chronological arrival.

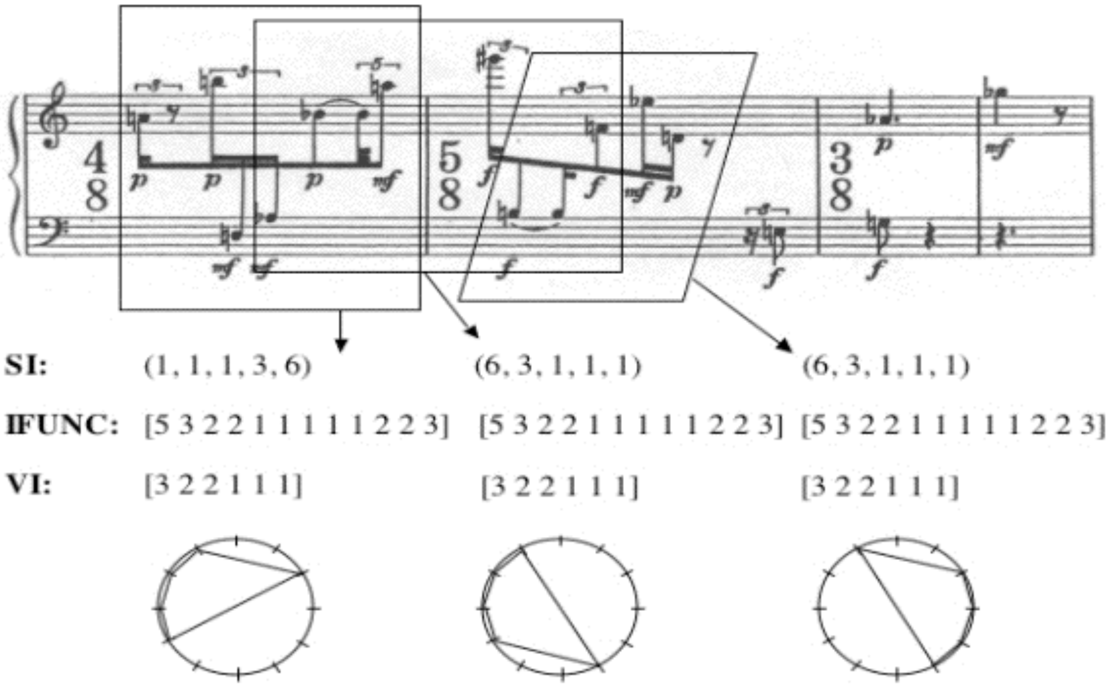

FIGURE 6. First measures of Stockhausen's *Klavierstück III* with a beginning of segmentation process by imbrication.
The segments, or regions, are described with the help of the circular representation and accompanied by the indication of the main three set-theoretical invariants, i.e. SI (intervallic structure, counting the intervallic distance between two consecutive notes), IFUNC (Lewin's Intervallic function, counting the multiplicity of occurrence of each interval, from the unison to the major seventh, within the chord) and VI (Forte's Interval Vector; counting the multiplicity of occurrence of each interval, from the unison to the triton, within the chord).

The following Figure (see Figure 7) represents the transformational network of the *Klavierstück III* in the analysis by Lewin, which we show by adding the circular representation in order to make the structural relations between the different pentachords more evident.

The notation for the pentachordal forms and the transformations are borrowed from Lewin's ones. Hence, the transposition of the pentachord P by n semitones, corresponding to the transformation $T_n(P)$, is written as P_n. The same notation is used for the transpositions of the symmetric pentachord p. The transformations P_6 and p_6 correspond, for example, respectively to the triton transposition of the pentachord P and of its symmetric $p = J_0(P)$. Note that all axial symmetries J_n which fix the chromatic tetrachord contained in P or p can be formally expressed as a transposition T_n of the basic J_0 inversion (which transforms P into p). For example J_6 indicates the composition of the J_0 symmetry with the T_6 triton transposition. One finds in this way the basic axiom of the transformational approach, i.e. the possibility of replacing the concept of *interval* in a given GIS (Generalized Interval System)[9] with the (generalized) transposition operation. Moreover, as the Figure 7 clearly shows, within the corresponding transformational network it is possible to find regions having the same arrow configurations. This enables to establish a 1-to-1 correspondence between regions of the network, the so-called "strong isographic relation", and to explicitly compute the number of strong isographies associated with a given transformational network.[10] We analyze in the next two sections the consequences of this change of perspective with respect of the notion of mental representation and, more generally, the epistemological and philosophical implications of this transformational paradigm in music analysis.

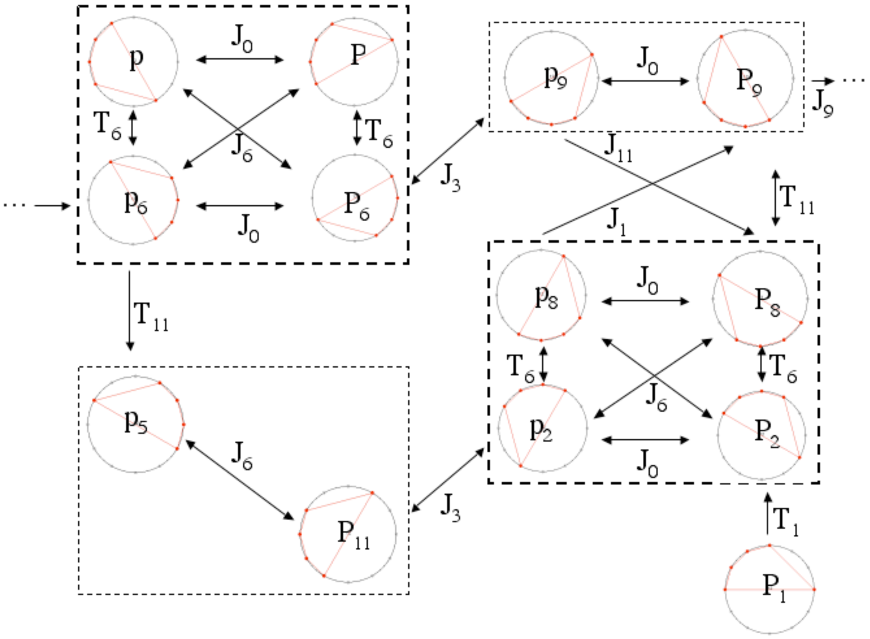

FIGURE 7. Transformational network of Stockhausen's *Klavierstück III* (after the analysis by David Lewin).
The network generates a conceptual relational space, where the different pentachords, which are geometrically represented by means of the circular representation, are related by transpositions or inversions.

9. TRANSFORMATIONAL ANALYSIS AND MATHEMATICAL PSYCHOLOGY

We strongly believe that the fact of coupling an algebraically formalized geometrical approach, such as the transformational one, with a computational perspective has some crucial theoretical implications for cognitive sciences and mathematical psychology.

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[9] We cannot enter, unfortunately, in the mathematical aspects of the GIS structure. Intuitively, a GIS is a set (which Lewin interestingly calls a "space") and a group of intervals acting on the space in a simply transitive way. See Andreatta [2012] for a formal definition of a generalized system within a more general philosophical discussion on the American music-theoretical tradition.

[10] By using category theory one can elegantly compute this number after formalizing strong isographies as a special type of limit within the functorial theory of denotators developed by Mazzola [2002]. See Mazzola and Andreatta [2006] for the first attempt at formalizing transformational theory in a categorical way.

One simple way to have the intuition of this change is to compare the transformational approach in music analysis with some mathematically-oriented directions in developmental psychology and cognition, such as Halford and Wilson's neostructuralistic approach [Halford and Wilson, 1980], Macnamara's and Reyes's logical foundation of cognitive sciences [Macnamara and Reyes, 1994], Ehresmann and Vanbremeersch's Memory Evolutive System Model [Ehresmann and Vanbremeersch, 2007] and Phillips and Wilson's Categorical Compositionality [Phillips and Wilson, 2010]. From an epistemological point of view, transformational analysis provides an instantiation, in the music domain, of Gilles-Gaston Granger's articulation between the "objectal" and the "operational" dimensions [Granger, 2004]. This duality was considered by the French epistemologist as the foundational basis for the very notion of "concept" in philosophy.[11] From the perspective of developmental psychology, among the three problematics which – according to the psychologist Olivier Houdé – mark the renewal of Piaget's genetic epistemology, category theory occupies a central place [Houdé, 1993]. Differently from the structural approach which Piaget developed starting from his *Essai de logistique opératoire* (1949) and which also constitutes the conceptual framework of his researches on the "*abstraction réfléchissante*", category theory introduces, according to Houdé, a new element in the operational thinking. Morphisms enable to take into account an "aspect of logical-mathematical cognition which does not proceed from the transformation of the reality (operations and grouping of operations) but which takes into account the simple relational activity [*mise en relation*]".[12]

This interpretation of the categorical approach explains, we think, a fundamental aspect of transformational analysis, which is the interplay between the notion of transformational progression and that of transformational network that we have described in the case of Lewin's analysis of Stockhausen's *Klavierstück III*. In a progression, transformations follow according to an order which respects the chronological unfolding of the piece. The operational logic remains anchored to a temporal component which, as in the case of the piece by Stockhausen, does not seem to capture the underlying musical logic as it is perceived by the listener. The main point is that in a transformational network the "operational logic" is created by the analyst by means of putting in relation objects with morphisms in an abstract space of potentialities. As stated by Lewin, the transformational network "shows a certain abstractly structured space of possibilities through which the piece moves, but it also shows how the abstract structuring is suggested and bounded by actual transitions within the progress of the piece itself" (Lewin 1993, p. 36). By paraphrasing Lewin's conclusion, in the case of transformational progressions we are at a given time of the narrative process of the piece, whereas in the case of an abstract network we are instead in one well defined point within the space created by the piece.

The question which remains open is the real "perceptibility"[13] of the relational conceptual space associated to a given musical piece, independently of the special case of the pentachordal transformational network underlying Stockhausen's *Klavierstück III*.

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[11] See, in particular, the article "Contenus formels et dualité", reprinted in Granger [1994].

[12] See Houdé [1993].

[13] An ongoing project between the Musical Representation Team at Ircam and a research group of the Schulich School of Music at the McGill University, directed by Stephen McAdams, is currently evaluating the perceptual relevance of Lewinian analysis of Stockhausen's *Klavierstück III*.

10. TOWARDS A PHENOMENOLOGICAL STRUCTURALISM IN "MATHEMUSICAL" REPRESENTATIONS.

It is probably too early to evaluate the epistemological consequences of a "paradigmatic change" in music analysis, as the transformational approach has been characterized in comparison to the previous approaches in music theory and analysis. Nevertheless, we can easily describe some potential interesting future research directions. Concerning the very nature of "space" in music, apart from the spatial programming paradigm, which has – as we have suggested – interesting applications in music, there are some intriguing connections between transformational analysis and Husserlian phenomenology, in particular in its relationships with mathematical idealities and physical reality.[14] Many questions raised by Luciano Boi in the presentation of the first part of *Rediscovering Phenomenology* [Boi *et al.*, 2007] focused on spatiality and phenomenology of perception, exceed the case of visual perception. In particular, in the case of the spatial organization of musical structures proposed by the transformational approach, the question arises concerning the relation between perceived forms and cognitive activity. This relation equally asks for an analysis of the local/global articulation within a given transformational network. We can establish, in this way, a very surprising connection between transformational analysis and the category-oriented model of the Memory Evolutive System by Ehresmann and Vanbreemersch.[15] This model offers an interesting example for approaching the complex hierarchical nature of human cognition from a categorical perspective by also taking its compositionality character into account. As somehow emphatically stressed by Steven Phillips and William H. Wilson, "Category theory offers a re-conceptualization for cognitive science, analogous to the one that Copernicus provided for astronomy, where representational states are no longer the center of the cognitive universe – replaced by the relationships between the maps which transform them" [Phillips & Wilson 2010, p. 1].

Music, and more precisely mathemusical research, represents a way of finding a conciliation between structuralistic tradition and Husserlian phenomenology, two approaches which are usually considered as antagonistic.[16] We have therefore proposed the term "phenomenological structuralism" for such a philosophical orientation in mathematical music theory and computational musicology.[17] One can propose this hypothesis starting, for example, from the writing by Ernst Cassirer [1944], whose algebraic considerations on the melody are directly inspired by Felix Klein Program in geometry [Klein, 1872] and whose thinking is also largely influenced by Husserlian phenomenology [Choi, 2009].

A philosophical discussion which seems to find a natural application to the field of transformational analysis is proposed by Jocelyn Benoist in the chapter of the already quoted *Rediscovering phenomenology* book [2007] on the phenomenological relevance of category theory. In fact, if one of the aspects characterizing the phenomenological approach is the attention toward the dynamics of conceptual intuition, the way in which

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[14] See L. Boi *et al*. [2007].

[15] The first musical applications of the model of Memory Evolutive Systems were presented and discussed during several sessions of the 11th season of the MaMuX Seminar at Ircam. The topic constitutes the object of the ongoing PhD thesis by J. Mandereau (University of Pisa/University Paris VI).

[16] In doing that, we have been crucially influenced by Jean Petitot's approach aiming at rediscovering the deep connections between structuralism and phenomenology. See, in particular, Petitot [1988, 1994, 1999, 2009].

[17] See Andreatta [2012]. The same concept was used by Jean Petitot, although in a different context, with reference to Jakobson approach in structural linguistics. This aspect was discussed in details by Petitot in his morphological reading of the genealogy of structuralism [Petitot, 1999].

David Lewin stresses the construction process within a transformational analysis – as exemplified in Lewin [1993] – suggests the possibility of coexistence of a phenomenological approach and a structural investigation in the domain of mathematical representation in music. In the same way as "Husserlian phenomenology of mathematics is structural, since it fixes itself on the invariants […] of which it makes the heart of the mathematical objectity [*objectité mathématique*] as formal objectity"[18], transformational analysis is phenomenological by being at the same time structural, since the group of transformations acting on the musical space is systematically confronted to the perceptive process which is peculiar to the subjectivity of the analyst. Starting from reflections of mathematicians on the phenomenological account of contemporary mathematics, and comparing these authors with other more epistemological orientations on the cognitive aspects of the phenomenological method, researchers in mathematical music theory, computational musicology and music informatics might find the way to constitute a new conceptual space within which some mathematical problems raised by music do have important perceptual implications and open perspectives enabling to enrich and renew the philosophical quest. This would surely lead to a better understanding of the geometric character underlying the complex notion of mental representation.

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[18] See J. Benoist [2007].

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