Physical study of double-reed instruments for application to sound-synthesis

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Physical models for most reed instruments have been studied for about 30 years and relatively simple models are enough to describe the main features of their behavior. These general models seem to be valid for all members of the family, yet the sound of a clarinet is distinguishable from the sound of a saxophone or an oboe. Though the conical bore of an oboe or a bassoon has a strong effect over the timbre difference, it doesn't seem to be a sufficient explanation for the specific character of these double-reed instruments.

The main hypothesis currently being developed is related to the geometry of the mouthpiece, and its effect on the flow inside the reed. Numerical simulations of a model based on this hypothesis show that it can introduce quantitative differences capable of explaining the behavior of double-reed instruments.

A more realistic model will take into account current experiments and measurements of the flow and the reed's dynamic behavior. Preliminary observations seem to be consistent with the hypothesis being tested and the mathematical model that we describe.

I. INTRODUCTION

Woodwind instruments of the reeds family are driven by a common principle: the oscillation of an air column is maintained by the intermittent flow running through the reed, which works as a valve. Such a general formula in fact does not explain the timbre difference between a double and a single-reed instrument. Given the similarities, one could expect that a model built for a single-reed would also work for a double-reed by adjusting its physical dimensions. This is not true, and more profound changes have to be performed in order to transform a single-reed model into a double-reed one.

For instance, one could argue that in a double-reed instrument two oscillators are necessary instead of one. Nevertheless, reed motion observations have shown that, in usual conditions the two reeds oscillate synchronously, so that the reed's slit width can be described by duplicating the coordinate of a single oscillator (see section IV).

A well-known and crucial factor in the sound timbre is the instrument bore: on one hand the frequency is controlled by it, on the other hand cones have all harmonics, while cylinders only have odd harmonics. Moreover, woodwind instruments usually have *weak* reeds, which are driven instead of driving the air column oscillations. However, the bore cannot account for the whole difference: there are instruments, such as the saxophone that use a single-reed coupled to a conical bore, and whose sound is immediately recognizable from that of an oboe. Moreover, oboists and bassoonists know that subtle tuning of the reed affects the sound independently from the bore profile.

We thus turn ourselves to the embouchure. The principle does not change much from the clarinet to the oboe, but its geometry does. The present work explores this difference through a physical model, and evaluates through observations of reed oscillation some hypothesis made in the model.

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II. MATHEMATICAL MODEL

A. Embouchure

In all reed instruments, the reed itself behaves as an oscillator driven by the pressure difference between its inner (p_r, reed pressure) and outer sides (p_m, mouth pressure). A simple approach to model such a system is the harmonic oscillator, which is equivalent to substituting the elastic reed by a mass/damper/spring controlled valve.

$$m_s \frac{\partial^2 z}{\partial t^2} + r_s \frac{\partial z}{\partial t} + k_s(z - z_0) = p_r - p_m \tag{1}$$

where the mass (m_s), damping factor (r_s) and stiffness (k_s), are given per surface unit.

The same pressure difference that drives the reed creates a flow through its opening. As proposed by Hirschberg [1] the flow is restricted to a smaller section than the reed, due to the *Vena Contracta* effect. The ratio of the flow to the duct section (α) is theoretically 0.5 for an infinitesimal width of the reed and a potential flow. In practice this coefficient varies from 0.6 [2] to 1 [3].

We will restrict ourselves to a quasi-static and non-viscous description, so that the we can apply the Bernoulli formula across the reed:

$$q = 2\gamma z l_r \alpha \sqrt{\frac{2}{\rho}(p_m - p_r)} \tag{2}$$

The factor $2z$ is the distance between the two reeds and l_r the reed's slit length. γ is introduced to take account of the geometry of the reed, which is different from a rectangle. $2\gamma z l_r$ is then the effective section of the flow.

The main difference between the clarinet and our double-reed model is that there exists a difference between the pressure at the beginning of the reed (p_r) and the acoustic pressure at the beginning of the bore. This difference is due to the geometry of the canal downstream of the reed, which induces perturbations in the flow.

In fact, in a clarinet the jet is formed at a narrow section region and, as it flows inwards, it goes through rapidly increasing duct sections (see fig. 1). As this happens, it becomes a free jet which, due to its high Reynolds Number, is unstable, so that it eventually breaks out by turbulence. Its kinetic energy is completely dissipated and we assume there's no pressure recovery as the flow stops.

On the other hand, for an oboe or a bassoon the duct section increases smoothly from the tip of the reed until nearly the bore ending [4]. Although the flow inside the reed canal is not well known at present (PIV velocity field measurements are being prepared) it seems reasonable to suppose that the flow is not separated from the walls (except on a short length just after the reed input) but guided by the duct.

A hypothesis about this flow assumes that there is an incomplete head loss, induced either by jet reattachment, which leads to visco-thermal dissipation due to wall interaction, by formation of turbulent flow or by further constrictions in the duct [5]. In either case, under certain simplifying hypothesis [6], the dissipation can be considered proportional to the square velocity of the flow:

$$p_r = p_b + \frac{1}{2}\rho\Psi\frac{q^2}{S_{ra}^2} \tag{3}$$

The strength of this dissipation factor can be controlled by the coefficient Ψ, which for now is empirical but expected to become understandable and measurable after more extensive measurements.

B. The bore

Assuming a linear character of acoustic propagation inside the bore, it can be described by its input impedance (Z), which relates the frequency components of the pressure (p_r) to the flow (q) at the beginning of the bore. The simulations are carried out in the time domain, so the bore is described in terms of incoming and outgoing pressure waves, which can be obtained from the former variables (p_r and q) and transformed using the reflection function:

$$p^- = r * p^+ \tag{4}$$

An arbitrary bore profile raises several problems, mainly due to changing of the conicity along the bore [7]. Though there are several approaches to this problem (ex. [8]), the bore is not the primary issue of this work. We will for now restrict ourselves to cylindrical bores, also because they provide results of easier interpretation as the reed is concerned.

A simple model of a cylindrical bore is accomplished by a Gaussian function centered at about ($2\frac{L_b}{c}$), the time of acoustic propagation from the reed to the bore ending and back to the reed (see fig. 2), and the width of the Gaussian peak is a measure of visco-thermal losses on the bore [9].

III. IMPLEMENTATION

For a cylindrical bore, the two partial waves p^+ and p^- are obtained by the formula:

$$p_b = p^+ + p^- \tag{5}$$

$$\frac{\rho c}{S_b} q = p^+ - p^- \tag{6}$$

where ρ is the air density, c the speed of sound and S_b the bore section area. The first factor in equation (6) is the bore characteristic impedance, also referred to as Z_0.

From (5) and (6) we extract a new relation which, together with equations (1-3) rearranged and discretized by an Euler method, will constitute the set of equations to be solved for each sound sample:

$$p_b - Z_0 q - 2p^- = 0 \tag{7}$$

$$q^2\left(1 + \Psi\frac{\alpha^2 l_r^2 z^2}{S_{ra}^2}\right) - 2\frac{p_m - p_b}{\rho}\alpha^2 l_r^2 z^2 = 0 \tag{8}$$

$$p_m - p_b - \Psi\frac{\rho}{2}\frac{q^2}{S_{ra}^2} + z\left(\frac{m_s}{(\Delta t)^2} + \frac{r_s}{(\Delta t)} + k_s\right) + \frac{m_s}{(\Delta t)^2}(z_{t-2} - 2z_{t-1}) - \frac{r_s}{(\Delta t)}z_{t-1} + k_s z_0 = 0 \tag{9}$$

In this set, p^- is calculated first, using the relation (4), and is thus a parameter in equation (7). In the relation (4), we notice that for any reflection function $r(t) = 0$ for non-positive t, due to causality. Therefore we only need to know the past values of p^+, and consequently of p_b and q. All variables with no index are taken at time t.

The set of equations does not have an analytical solution, so we use a Newton-Raphson iterative method to solve it, with the solution found at the former time as the initial guess.

In practical simulations, the model requires the estimation of many parameters which cannot be measured directly on an instrument (for instance, m_s, k_s or r_s). We started with the parameters existing in the literature [10], [4] and adjusted them in order to have realistic oscillatory regimes.

IV. EXPERIMENTAL PROCEEDING

A simple device is used to obtain measurements of reed displacement and pressure at the beginning of the bore. It substitutes the instrumentist, providing a more accurate and continuous way of controlling the parameters applied to the instrument.

The device consists of an *artificial mouth* which is a 90^{o} angle tube, on which we created windows (covered with transparent plastic), in order to allow the observation of the inside of the tube from all sides (fig. 3). The embouchure of the instrument is placed inside this tube with the reed facing the windows. On the other side, the tube is connected to a reservoir which is fed with constant pressure compressed air. The reservoir renders the flow uniform. The static pressure is measured by a digital manometer upstream of the instrument.

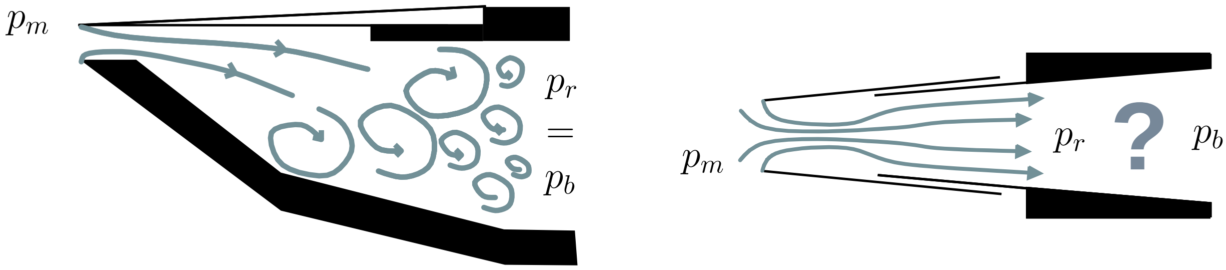

FIG. 1: Flow inside clarinet and oboe embouchures

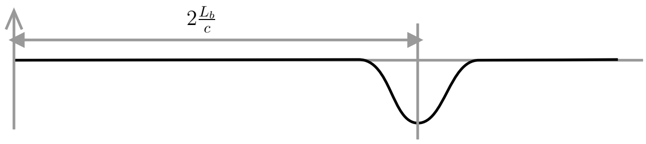

FIG. 2: Reflection function for the cylinder

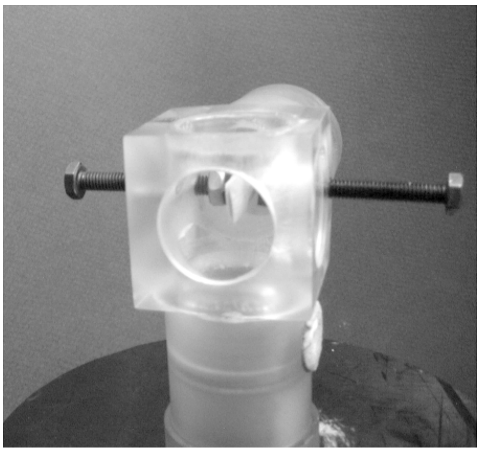

FIG. 3: *Artificial mouth* used in experiments

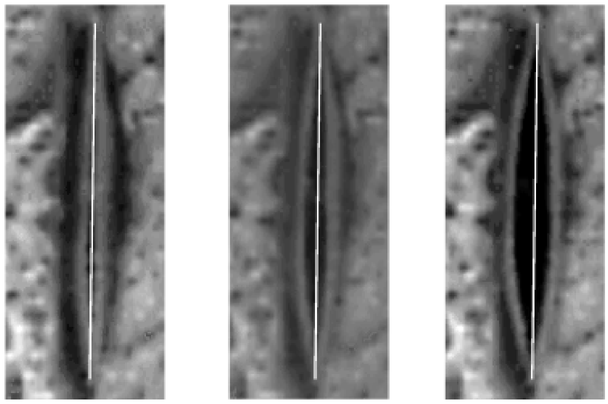

FIG. 4: Reed motion. Form left to right: almost closed, average position and maximum width, just after opening.

The reed tip is illuminated by a stroboscope which is synchronized to the sound radiated by the instrument. The instrument frequency has to be manually synchronized to the camera's refreshing rate.

The images are transferred to a computer in order to be processed. Post processing consists in finding the border between the reed and the slit. The fact that the reed is illuminated during a strobe flash while the slit appears as a black region on the pictures allows the use of automatic methods of recognizing the slit. Its area, largest and smallest axis are measured for each picture. The largest axis, averaged over the whole film corresponds to the constant slit length, measured in the physical reed. This is used to find out real dimensions on the picture.

Measurements were taken for two different oboe reeds, one plastic-made and one cane-made. For each of the reeds several film clips were recorded, ones with the reed freely oscillating, others using a clamping wire curled around a sponge, so as to replace the instrumentist's lips.

Direct observation confirms the former hypothesis that the reed motion is symmetric (fig. 4). It is worth saying that non-symmetrical motion can be achieved, but this does not correspond to usual modes used in musical performance.

V. SIMULATION AND EXPERIMENTAL OBSERVATIONS

As a starting point, it is interesting to consider a simple model which consists of making the mass and damping of the reed equal to zero, an approximation which is frequently made in clarinet models. Its interest is that there exists a single pressure-flow curve to which the system belongs at any time and which can be analytically described (fig. 5).

The exact portion of the curve that will be spanned by the system can only be known by running the simulation. It is possible however, by playing with the physical parameters (such as mouth pressure, impedance and reed stiffness) to change the range of variation on the curve. In fact, the system will have to oscillate around the decreasing slope of the curve, because only in this part does the reed supply the acoustic oscil-

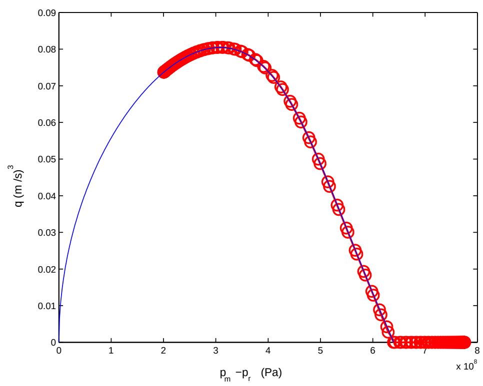

FIG. 5: Characteristic pressure vs flow curve for a clarinet-like instrument (line – theoretical curve, calculated analytically; circles – simulation samples).

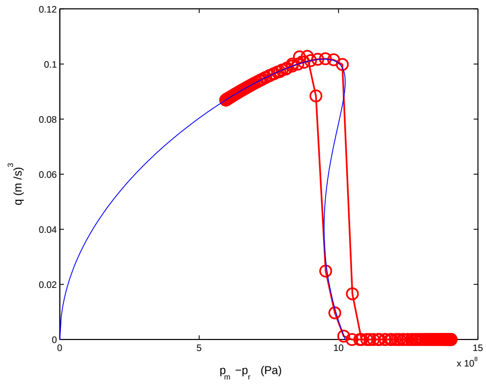

FIG. 6: Characteristic pressure vs flow for an oboe-like instrument.

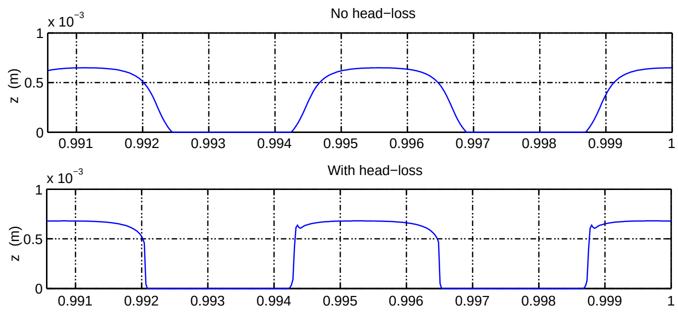

FIG. 7: Time evolution of the reed position (z) for low and high Ψ.

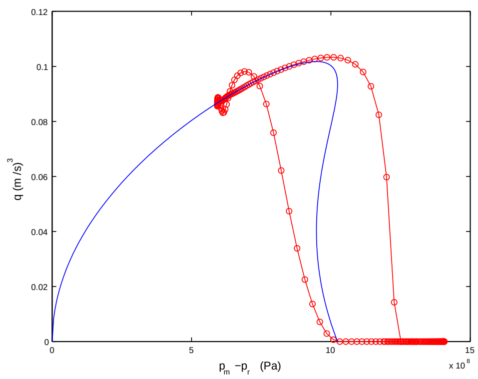

FIG. 8: Phase-space trajectory for a double-reed instrument with reed mass and damping (circles). Analytical curve for zero mass and damping (line).

lations in the bore with energy.

By increasing the coefficient Ψ (fig. 6), there are multiple possible flows for the same pressure. Assumptions on switching between different solutions are that the system remains on the same branch while it can, and changes to the nearest branch as soon as it cannot. This fact implies that the flow, and the reed opening have rather abrupt changes for a smooth variation of the pressure (which is the case for the given reflection function). This is visible on the flow and reed coordinate plots (fig. 7).

A similar observation can be made on measurements made on real reeds (fig. 11), which show that reed opening and closing are substantially faster on double-reeds than on single-reeds. For an example, our measurements can be compared with those made by J. Gilbert [3].

A. Reed resonance

A real instrument of course has a non-zero mass and damping, and its behavior deviates from the previously plotted curve as these parameters increase (fig. 8). For instance, mass and damping have the effect of smoothing out abrupt variations of the reed position. The figure shows how the system takes more time (more samples) to travel from open to closed position.

Another consequence is that mass and elasticity together constitute a harmonic oscillator with its own frequency. While for playing frequencies close to the reed proper frequency the final result is rather complicated, with beating and quasi-periodic oscillation phenomena, for lower frequencies, the reed oscillation shows out more distinctively after a reed opening (fig 9), and gets fainter till the end of the opening time. This effect is called *reed resonance*.

Although this effect is quite visible on the time variation of the reed position and flow, the effect over the pressure wave is not perceptible on the graphs (fig. 9). However, recall that these graphs show the variation of pressure at the beginning of the bore, rather than the radiated sound, which is affected by high-pass radiation filters. Even so, differences can be heard while listening to the pressure wave.

Figure 10 (top) shows our measurements of the reed position, taken from a free (unbitten) reed. Tight oscillations appear in each period, and are more pronounced than on figure 9. Their structure seems nevertheless more complex than that

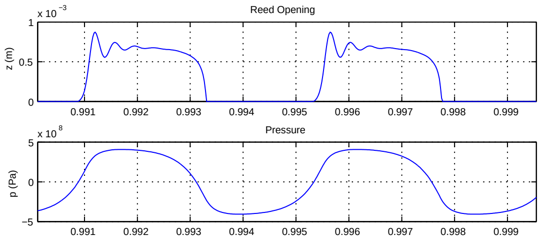

FIG. 9: Reed opening and pressure waves corresponding to fig. 8.

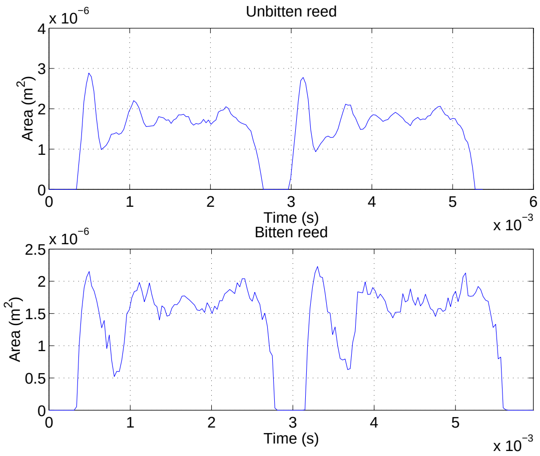

FIG. 10: Opening area measurements on a real unbitten and bitten reed.

of a harmonic oscillator, indicating that probably the reed's one-dimensional model is inaccurate.

B. Effect of biting

Figure 10 depicts two reed area measures. The biting is achieved through a rather crude replacement of the lips described above. The material properties of the sponge are probably very different from those of the lips, but the graphs give us some insight of the behavior of the reed.

Notice that the bottom graph is flattened at the top, the reed not opening as much as without biting. The structure remains nevertheless the same: a wider oscillation at the beginning and fainter ones following the first. The average position remains more or less the same.

The effect is not easy to simulate on the model, because it is not clear which parameters must be changed in order to achieve a similar result. A guess is to increase the stiffness and the mass together, in order to maintain the reed resonance frequency and the damping as well, but the flattening effect is not achievable, probably because the lip requires a much more complex model, which accounts for different rigidity at different openings.

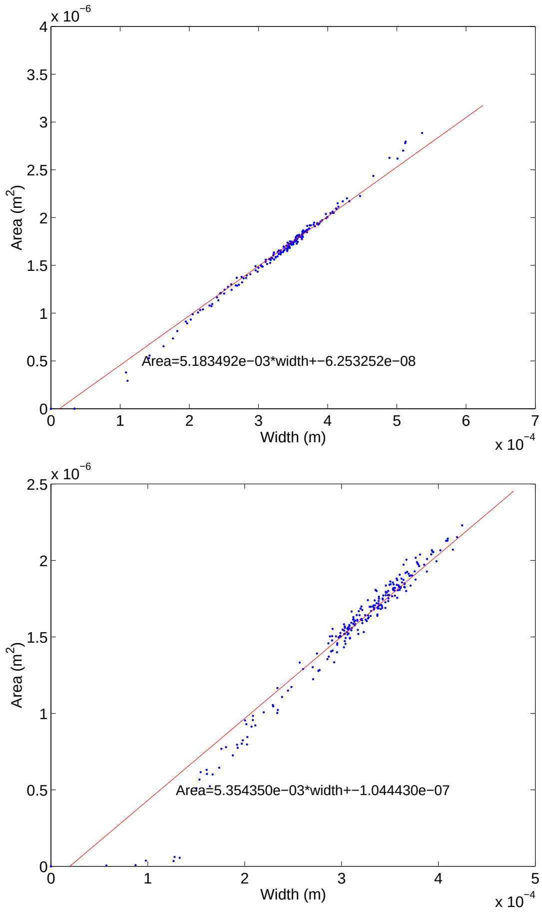

FIG. 11: Opening area measurements on a real unbitten and bitten reed.

C. Reed slit model

An important feature of the image analysis is that it gives an idea of the slit geometry variation during a period, and not only of a single coordinate. For instance it is possible validate the model of linear area/width dependency and determine the proportionality coefficient.

By linear regression (fig. 11) this coefficient is found to vary between 5.3 and 5.9 mm, which means that the γ coefficient should be between 0.8 and 0.9, supposing a linear correlation between the two variables. In fact the relation is more non-linear for increasing biting strengths.

D. Other effects

On the observations, the time the reed remains closed is less than half a period for low frequencies, and usually does not vary with the period length. In the simulations we have typical closing times of half a period, independent of frequency. This is an effect of the bore profile, which is conical in the observations and cylindrical on the simulations. In fact, in a conical bore the fraction of the closing time corresponds to the ratio of the truncated length of the cone and its total length [11], [4].

As also observed by Gokhshtein [12], the character of the reed resonance (frequency and shape) is independent of the total period length. A possible explanation is that this is a consequence of the impulse response of the reed, which is truncated earlier for shorter period times.

VI. CONCLUSION

A model based on a hypothesis of head-loss inside the reed has been studied and simulated in the time domain. Results of such simulations are compatible with experimental data, showing that this hypothesis provides a possible explanation to the differences between single and double reeds.

Simulations which take account of mass and damping are coherent with our experimental measurements. At the same time these observations appear to indicate that the behavior of the reed is more complex and needs a multi-dimensional model to be described.

Further measurements, such as a more complete study of pressure and displacement time-variations and an extensive description of the flow inside the reed are being prepared and hopefully will allow us to propose a complete model describing the main features of the double-reed behavior.

Finally, with a complete model of the embouchure and the current knowledge about the modeling of waveguides, we hope to achieve an instrument model capable of reproducing the sounds of double-reed instruments.

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