Observers of a nonlinear neutral system modelling a musical brass instrument*

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Abstract

A model of a trumpet-like instrument is described by a nonlinear neutral system controlled by the mouth pressure. The output is the pressure at the end of the pipe. A nonlinear state observer of this system is built. Local asymptotic stability of the error estimation is proved using a Lyapunov-based analysis and illustrated by simulations.

Keywords

Nonlinear neutral system, observer, Lyapunov function, musical instrument.

1 Introduction

The control of virtual musical instruments is as difficult as that of real instruments. Inverse problem techniques can help to capture skilled musicians'gestures, by recovering parameters and inputs from a target sound [1]. This paper solves a part of the global problem, building an asymptotically stable observer of the instrument state.

2 Simple model and neutral state-space representation

The trumpet-like instrument considered here couples an ODE (a valve including the mechanics of the lips) to a PDE (acoustic pipe ended with a real passive impedance) through a static nonlinear function (Bernoulli relation on the jet). Solving the PDE, the overall system can be described by the nonlinear neutral state space representation $\dot{x}(t) = f\big(x(t), x_3(t-\tau), \dot{x}_3(t-\tau), v(t-\frac{\tau}{2})\big)$, with input $v = [p_m, \dot{p}_m]^T$, output $y = [0, 0, 1+\lambda]x$, $(\lambda \in]-1, 1[)$, state $x(t) = [\xi(t) - \xi_e, \dot{\xi}, p^+(t)]^T$, where

*This work is supported by the CONSONNES project, ANR-05-BLAN-0097-01

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τ, p_m, ξ ($/\xi_e$), p^+ denote respectively the delay proportional to the length of the pipe, the mouth pressure, the distance between lips (/at equilibrium) and the ingoing pressure at the entrance of the pipe. Moreover, $f_1(x,y,z,v) = [0,1,0]x$, $f_2(x,y,z,v) = -\omega^2 x_1 - \alpha x_2 + \beta_0 x_3 + \lambda\beta_0 y + \beta_m v_1$, and, if $x_1+\xi_e > 0$, $f_3(x,y,z,v) = \left[v_2 - 2\lambda z + \frac{2\,x_2\left(v_1 - x_3 - \lambda y\right)}{x_1+\xi_e}\right]\left[1+\mu\frac{x_3-\lambda y}{(x_1+\xi_e)^2}\right]^{-1} + \lambda z$, and if $x_1+\xi_e \leq 0$, $f_3(x,y,z,v) = \lambda\, z$ where β_0, β_m, α, ω, λ, μ are positive coefficients depending on the lips and the pipe characteristics.

3 Observer, local robustness, and simulation results

An extended Kalman filter type observer is $\dot{\widehat{x}}(t) = f\left(\widehat{x}(t), \widehat{x}_3(t-\tau), \dot{\widehat{x}}_3(t-\tau), v(t-\frac{\tau}{2})\right) - \Lambda_1\left(y(t) - \widehat{y}(t)\right) - \Lambda_2\left(y(t-\tau) - \widehat{y}(t-\tau)\right)$ where the 3×1 gain matrices Λ_1, Λ_2 are tuned so that the linearized error equation $\dot{e}(t) = \widehat{A}(t)\, e(t) + \widehat{B}(t)\, e_3(t-\tau) + \widehat{H}(t)\, \dot{e}_3(t-\tau)$ is asymptotically stable where $e = x - \widehat{x}$ and $A(X,Y,Z,V) = \frac{\partial f}{\partial X}(X,Y,Z,V) + \Lambda_1[0,0,1+\lambda]$, $B(X,Y,Z,V) = \frac{\partial F}{\partial Y}(X,Y,Z,V) + \Lambda_2\,(1+\lambda)$ and $H(X,Y,Z,V) = \frac{\partial F}{\partial Z}(X,Y,Z,V)$ are evaluated at $(X,Y,Z,V) = \left(\widehat{x}(t), \widehat{x}_3(t-\tau), \dot{\widehat{x}}_3(t-\tau), v(t-\frac{\tau}{2})\right)$.

The tuning of Λ_1, Λ_2 and the proof rely on the four following key points: Λ_2 can be tuned such that $\widehat{B} = 0$; $\|H(t)\| \leq |\lambda| < 1$; let $\chi > 0$, then Λ_1 can be chosen such that $\widehat{A}(t)$ takes the form $\widehat{A} = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ -\omega^2 & -\alpha & 0 \\ \hline \widehat{A}_{31} & \widehat{A}_{32} & -\chi \end{array}\right)$, ensuring that e_1, e_2 are decoupled from e_3; $\exists\kappa > 0$, $\eta > 0$ such that, for every solution of the error equation, $e_1^2(t) + e_2^2(t) + e_3^2(t) + \int_{t-\tau}^{t} \dot{e_3}^2(s)ds \leq \kappa e^{-\eta t}\left(e_1^2(0) + e_2^2(0) + e_3^2(0) + \int_{-\tau}^{0} \dot{e_3}^2(s)ds\right)$ (see [2] for a similar proof). Simulation results are given in Fig. 1.

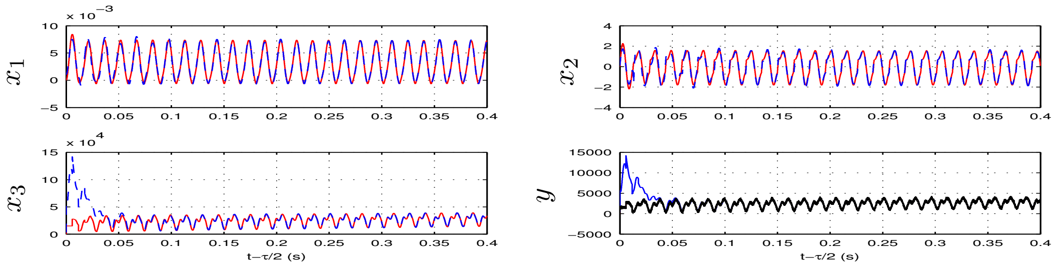

Figure 1: Simulations of the system (-) and of the observer (--) from the measured output y (-) with $\alpha = 150s^{-1}$, $\chi = 160s^{-1}$, $\tau = 2/340$s, $p_m = 1.5e4Pa$ (Heaviside shape), $\omega = 535s^{-1}$, a deviation on initial conditions $\widehat{x}(0) = [1e-3, 1e-1, 0.2\,p_m]^T$ and additional gaussian noise on the measured output with variance $\sigma = 0.01p_m$.

References

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