Abstract

The objective is the design of a software capable of carrying out an analysis of a musical score, not in keeping with the precepts of an a priori musical theory, but on the contrary as autonomously and neutrally as possible. For this purpose, inductive mechanisms have to be integrated to the system. A cognitive metaphor – in particular the procedural vision of induction proposed by Holland et al. (Holland, Holyoak, Nisbeth, and Thagard 1989), with mental model as a semantic network featuring multi-weighted hypotheses conflicting and corroborating each other –, overriding logical or probabilistic inconsistencies, efficiently answers this problem. Theoretical inquiries about induction have shown the fundamental relationships between induction and analogy. This is even more pertinent in our context, since analogy, as has been shown by either cognitive or musicologic studies, is the core mechanism for musical entities emergence. Our vision of musical analysis through systematic induction of analogies integrates melody, harmony and form into a unified framework and suggests new kinds of analysis that could grasp those music – non-occidental, contemporary, electro-acoustic, improvised – not understood by traditional analyses.

2 Induction: A natural approach of music analysis

Some say that today’s techniques of music analysis are sufficient for understanding the essence of music. Hence Nicholas Cook "see no intrinsic merit in the development of ever more rigorous and sophisticated analytical methods: though there are areas which are analytically under-developed (early music is an important one), [...] our present analytical techniques are rather successful." For him, the idea would be more to combine points of view conveyed by different techniques than inventing new ones.

The trouble is, those areas, conceded by Nicholas Cook, in which most of those traditional analytical tools are fairly lost, consist in fact of any style of music that was not explicitly taken into account by these tools when conceived. As a matter of fact, these analytic tools, as they implicitly describe the characteristics of a certain style of music, may themselves be considered more as an analytical result – a method to retrieve some style aspects in every piece – than as a pure analytical process. Moreover, even for the more canonical musical works, these kind of analysis reduce their content instead of explicating their specificity.

This idea was already formalized, in a very general epistemological point of view, in the seventeenth century, by the philosopher Francis Bacon. "There are, and can be only two ways of searching into and discovering truth":

- "The one flies from the senses and particulars to the most general axioms, and from these principles, the truth of which it takes for settled and
immovable, proceeds to judgment and to the discovery of middle axioms. And this way is now in fashion.” In musical context, these general axioms are those traditional music theories, constructed, like scientific theories, hypothetically.

- “The other derives axioms from the senses and particulars, rising by a gradual and unbroken ascent, so that it arrives at the most general axioms last of all. This is the true way, but as yet untried.” This may take into account the particularity of any phenomenon, and produce general knowledge instead of simply needing it. This approach, vigorously defended by Francis Bacon, is called *inductive*, because it aims at producing knowledge from phenomena.

The inductive paradigm has been relevantly criticized by modern epistemology, because there cannot be a *reduction* of scientific knowledge in terms of observed phenomena. There must be hypothetic axioms somewhere. This objection, however, is not valid in the realm of communication processes, or semiotic systems (Nattiez 1990) if you prefer. The music itself, concretized in a score or a signal – the *neutral level* –, is the result of a *poietic* process – the act of composition –, and is now subject to the *esthetic* grasp of either the reader (musician or analyst) or the listener. As most of the abstraction of musical language seems to be – potentially, and mainly implicitly – reducible directly to our mere perception of it, music analysis, as a kind of perception, may profit from an inductive approach.

This “true way”, nowadays, has been tried in a musical context. Rudolph Reti has experienced an analytical methodology that studies the score very minutely, trying to understand each note in its context. He then proceed to a “gradual and unbroken ascent” from microscopic (motivic) to macroscopic (formal) level. “And the true structural dynamism of a composition, its form in the fullest meaning of the term, can be conceived only by comprehending as a concerted stream both the groups and proportions of its outer shaping and the thematic evolution beneath.”

This inductive approach is the only way to achieve a satisfying understanding of musical language. But this task is so complex and implies such an overwhelming combinatory that “Motivic analysis easily degenerates into a purely mechanical exercise in which the score is analyzed without ever really being read properly [...]”. The whole tendency of motivic analysis is to suggest that music is some kind of complicated cipher, and that the way to break the code is to stare at the score for long enough. It does not encourage sensitive listening. Hopefully, the use of computer, alleviating us of the mechanical exercise, may answer to this objection. But we need then to implement – and, before that, to model – these inductive mechanisms.

Reti himself was blamed for not proceeding to really objective analyses. He was indeed inclined to express implicitly his subjective esthetic of music. For this reason too, inductive mechanisms have to be explicated objectively.

### 3 Cognition: A natural modelling of inductive mechanisms

A long philosophical inquiry has tried, since Antiquity, to understand the phenomenon of induction. Aristotle, when trying to define the concept of induction, integrates it in a *logical* framework, by considering it as a kind of reverse of syllogism. This logical point of view has been fairly developed, especially during the XXth century, in particular with the inductive logic of Rudolf Carnap. It has been a failure, though, because induction, contrary to deduction, cannot be artfully reduced to some elementary and abstract axioms, and also because we cannot proceed to induction if we consider knowledge in the form of predicates or linguistic propositions. In a word, induction is not an abstract calculus, but a pragmatic process.

Although Aristotle formalized induction inside a logical framework, he kept in mind the important fact that induction is a natural and psychologic process that enables us to catch a general idea out of phenomena. This psychologic dimension of induction has been developed especially by David Hume (Hume 1748). He characterizes it as a kind of habit, and, more precisely, demonstrates its foundation on imagination. This description, however, is only partial because induction has to be rooted in a priori mechanisms, as said Immanuel Kant (Kant 1781). Charles Peirce (Peirce 1992) managed to formalized efficiently these ideas of imagination, and hence induction, with the help of graph logic, or, more generally, a network. Indeed, the connexionnism of a network of concepts – or semantic network – may be considered as a generalization of logic.

The inductive logic of Rudolf Carnap not only failed because of the obsolescence of logic, but also because of its foundation on a related paradigm, namely, *probability*. Leibniz invented the concept of probability in order to explicate the degree of certitude of uncertain knowledge in a mathematical framework. But, in any way we consider probability, either subjectively – by considering a universe of possible – or objectively – through statistical measurements –, it is a unidimensional quantity that has to be fixed for any hypothesis. In the connexionnism vision of knowledge as a semantic network, the idea of probability is given up and replaced by a set of several distinct quantities that take into account different aspects of knowledge relationships: degree of match, past experience award, support
from other hypotheses, etc.

The idea by Peirce of a network of concepts takes place in his pragmatist program, alleviating conceptual framework of useless paradigm and integrating new ideas describing psychological and effective realities. Today, such pragmatic ideas are echoed, in a way, by cognitive sciences. The cognitive point of view is of epistemological importance, because it explains our understanding by describing its nature. Cognitive approach of induction, especially by the collectif of AI researchers, experimental psychologists and philosopher Holland et al. (Holland, Holyoak, Nisbeth, and Thagard 1989), takes benefit from the conclusions of all these philosophical inquiries. They emphasize the need to consider knowledge as a semantic network, where the firmity of hypotheses, in conflict and corroboration each other, depends on those of parent concepts in the network. Holland et al. add the essential idea that induction is a temporal process, where hypotheses are constantly trying to explain the new observed phenomena.

4 Analogy: A natural mechanism of music perception

John Stuart Mill (Mill 1866) has shown that a lot of knowledge of particular facts, instead of being deduced from general concepts, are based upon the degree of resemblance between the considered phenomenon and a set of reference phenomena. This means that analogy is an essential cognitive mechanism. And we can suppose that even when we take into account general concepts, we also have to find the adequate concept by analogy between the considered phenomenon and the general concept. In any way, therefore, inference of knowledge about a phenomenon fatally needs an analogy with other phenomena — either other samples or an abstracted one. We may suppose that the choice between these two alternatives generally depends on the quantity of known analogs.

This is in fact what Leonard Meyer means in his theory of expectation. Indeed, he envisions music listening as a dynamic process. At any time, "music arouses expectations, some conscious and other unconscious, which may or may not be directly and immediately satisfied". The actual continuation, if not what was predicted, triggers emotion, because "emotion is evoked when a tendency to respond is inhibited". And the core idea is that these expectations are learned, because they rely in fact on the memory of past musical examples. Meyer, because he thinks that "embodied musical meaning is [...] a product of expectations", implicitly applies Mill’s point of view in a musical context.

The idea of analogy has been even more explicitly considered in the paradigmatic analysis methodology applied in music by Nicolas Ruwet (Ruwet 1972). Inspired by linguistic, Ruwet proposes an analysis of music which, through a research of repetitions, detects the different motives, their inner structuration and their global organization. However, Ruwet’s approach is far from achieving Reti’s ideal. Indeed, the linguistic metaphor, though productive, does not take into account the intrinsic specificity of music. Indeed, such an approach considers music as a monodic flow — or a superposition of monodic flows — but never as a polyphonic network of intricated flows. Moreover, the static linguistic idea of paradigm is totally contradictory to the dynamic musical idea of development. And this methodology cannot be implemented on computer unless criteria of detection of similarity be defined explicitly and a priori.

Lerdahl and Jackendoff’s analytic methodology shares with Ruwet’s the idea of a hierarchical music representation. They are sensitive to the idea of analogy and repetition — which they call parallelism — but recognize not to be "prepared to go beyond this", and to "feel that [their] failure to flesh out the notion of parallelism is a serious gap in [this] attempt to formulate a fully explicit theory of musical understanding". This is due to that fact that they rely on a static grammar, instead of a pragmatic study of inductive process. Moreover, a pertinent modelling of "parallelism" would express more freely through an associative network than a hierarchical tree.

Analogy has been implemented in artificial intelligence applications, in particular by Douglas Hofstadter (Hofstadter 1995). He agrees in a way with Holland and al.’s framework, in particular with the idea of a network of activated concepts. Through a drawing of multiple possible analogies between the different elements of the structure, Hofstadter’s software Copycat, whose aim is to analyze short sequences of letters, builds a semantic network of relations. In this analogy framework, we would like to add the other pivotal idea of Holland et al.’s modelling of induction, namely the procedural approach. Indeed music is a temporal object, as would say Husserl, and a natural way of explaining it is by considering its temporal effect in conscious, as in Leonard Meyer’s expectative approach.

5 kanthume theory of analogy

5.1 Principles

We have shown why a cognitive approach of musical analysis is of great importance, and why, along the temporal progression of music perception, it will consist mainly of a research of analogy between current instant and past ones. Our software, called kanthume, is a tentative of simulating this point of view. We share Ruwet’s idea that motives — of notes, but also of sets of notes, of motives, etc. — have to be found through

(Meyer 1956), p. 25.
(Meyer 1956), p. 22.
(Meyer 1956), p. 35.
(Lerdahl and Jackendoff 1983), p. 53.
the detection of their repetition, varied or not. But if this research has to be efficient enough such as to be able to detect motives hidden in a polyphonic flux, it is necessary to go beyond Reti’s hierarchical framework.

First, we propose to formalize any musical structure simply in terms of a motive, or a sequence of elements, which themselves may recursively be motives too. That is, every musical structure is ordered. It seems that relationships between musical entities are determined along two dimensions, whose basic relations are:

- relations of analogy, between two analogs.
- relations of concatenation, between lateral elements within a motive.

Now if we consider that motives emerge because of their repetition – even when they vary – then this can be possible only if:

1. Something in the beginning of the repeated motive triggers the idea of analogy with first motive: either a same value of a parameter for the same element of the two motives (pitch, duration, etc.), or a similarity of an interval between two elements in the two motives (here also pitch interval, onset interval, etc.).

2. The successive next musical items share some similarity with the corresponding ones in the first motive: mostly because of similarity of interval, but, why not, of similarity of an absolute parameter of one particular note. Each new element will be considered as the continuation of a repeated motive if it can be linked (particularly by an interval) to an element of its beginning and if this link has its analog in the first motive. This element to which it is linked can be called, metaphorically, the anchor of the new element.

5.2 Description

During analysis, music is considered incrementally in a chronological sense. At each step, new note of the score is considered. From current note \( n \), a series of intervals are drawn to all precedent notes within a short term area. For each of these intervals \( (m, n) \), the system finds the set of similar previous past intervals. For each of these similar intervals \( (i, j) \):

1. if \( i \) is the right note of an interval \( (h, i) \) which concludes a motive \( [\cdot, (h, i)] \), and if \( m \) is also the right note of an interval \( (l, m) \) which concludes another motive \( [\cdot, (l, m)] \), and if both sequences are analogous, then there is an analogy, or a sequencing, between the two extended motives \( [\cdot, (h, i)(i, j)] \) \( [\cdot, (l, m)(m, n)] \);

2. else if the two notes \( i \) and \( m \) are analogous, then there is an analogy between the two new motives \( [i(i, j)] \) \( [m(m, n)] \), where \( [i(i, j)] \) is the motive constituted by the simple note \( i \) and the interval \( (i, j) \);

3. if no analogies at all can be inferred between \( i \) and \( m \), then only a analogy may be drawn – if necessary – between the two simple motives \( [(i, j)] \) \( [(m, n)] \).

In this way, a network of analogies is drawn from the notes of the score. These analogies also emphasize the notes and intervals that belong to them. That is to say, the more a note, or an interval, belongs to numerous or big sequences, the more probably will it be a candidate analog.

Now each analogy is itself an interval whose two elements are its two analogs (that is, the two whole motives). The (multi-dimensional) value of this interval is called the analog-interval. When two analogies of the same kind have similar analog-interval, new analogies are triggered in the same way as for the similarity of previous intervals \( (i, j) \) and \( (m, n) \).

Our hypothetic claim would be that the whole set of links automatically inferred by this theory is sufficient to retrieve all the concept induced by traditional musical analysis, and, much better, all the understanding of music implicitly experienced by a simple listening of music.

The architecture of kanthume, that we will now describe, has been determined in order to fulfill this research of analogies.

6 kanthume architecture

6.1 The relationship network

The note object. Each note of the original score is represented as a note object inside the relationship network. The note object contains the value of the note parameters: basically, pitch, date and duration. The notes are inserted inside the relationship network, incrementally and in a chronologic order. The relationship network progressively digests the notes of each new instant of the score: that is, each time a new chord is added, new relationships propagate along the network, which enters a stable state before considering new musical events.

The note object also features pointers onto the different sequences and analogies to which it belongs, either as an analogous note, or as an element of an analogous interval.

The note parameter hash-tables. For each possible note parameter, a hash-table associates each parameter value with the set of the notes where this value holds (see Figure 1). The pitch hash-table is considered in two ways: in an absolute point of view as a correspondence between any pitch value and its occurrences, and
in a chromatic point of view as a correspondance between any pitch of the chromatic scale and its occurrences. The second point of view consists of considering the set of all the pitch value equal to the absolute pitch value modulo 12.

**The interval network.** Each time a new pitch or date value is added to the hash-table, the new value is linked to each possible old value (the values, not the events). For each of these sets of links, forming two interval networks (a pitch-interval network, and a time-interval network), is associated the value of the interval.

**The interval hash-tables.** As for note objects, two additional hash-tables, one for onset and one for pitch (see Figure 2), associate each interval value with the set of the links inside the interval network where this value holds. Once again the pitch interval hash-table can be considered following two points of view: the absolute one and the chromatic one.

**Note relationships.** Each new note is linked to its corresponding note parameter hash-tables. These hashtables, by definition, automatically detects the equality of the current note parameters with old ones. In our framework, we propose not to consider these hash-tables in a binary point of view. We prefer instead adding a similarity-distance that enables to consider not only equality, but also similarity of values, for each possible parameter. For each candidate note is associated an activation degree that consists of all these similarity-distances, plus the note support parameter (see paragraph 6.2). When activation exceeds a certain activation-threshold, an analogy is inferred between the current note and the activated one.

**Interval relationships.** The same is true for intervals. The trouble is, the comparison of every possible interval from current note with every possible other interval is of course a task that may explode for a long musical sequence. It is necessary, therefore, to limit the scope of the study of interval relationships. Concerning the choice of interval from the current note, two factors are taken into consideration: the time interval and the support (see paragraph 6.2) of the note at the other extremity of this interval, that induce two new distances, namely time-distance and support-distance. Once these intervals are chosen, the activation of related intervals (using two support-distances for each extremity) and the triggering of analogy follow the model of note relationship activation, this time using the interval hash-tables. For the comparison of absolute intervals, a new distance is added: the pitch-distance between the two high notes (or two low notes).

6.2 The analogy network

**The analogy object.** Any analogy relationship may be represented in general as a couple of two analogs. The analogy object also lists the parameters that are common to the analogs, and the amount of corresponding similitude. Finally, as for note object, it contains a list of pointers to higher-order analogies, in which it belongs as an analog.

**Support.** To any note or analogy is associated a dynamic parameter called support, equal to the number of analogies of any order that are constructed from it. This measure indicates the importance of present note or analogy, and plays a role in detection of new analogies: the more a note or analogy is supported, the more it will be used as an analog for a new analogy.

**Analogy inference.** When considering triggering new analogies, several parameters are taken into account: the distance of each analogous interval, the multidimensional degree of similitude between each candidate analogue, the support of the anchor (see point 2 of section 5.2). These different parameters are considered in parallel, that is, analogies are triggered if one parameter – or any collaboration between several parameters – is particularly significative. To this framework is added the constraint of a limited number of triggerings: in case of competition, only the most favourable candidates will be chosen. The different functions and threshold that control all these competitions – which exactly correspond to the competitive model of Holland et al. – can be edited by the user.

7 An example of analysis

In order to appreciate the musicological interest of such a framework, here is how kanthume analyzes the nine first bars of the fifth symphony of Beethoven, reduced for piano as in figure 3. Throughout this analysis, notes will be denoted by the number of their instant and their rank within the instant – from high to low – by a letter (a, b, c).

7.1 Instant #1

The value of the parameters (chromatic and absolute pitch, duration) of these three first notes are registered in their respective hash-tables. Interval parameters (interpitch and interonset) are registered too. Both three notes have same chromatic pitch (G), but since they are synchronized, they are not considered as analogous (because we only consider analogy with old notes). Idem for the two octave intervals.

7.2 Instant #2

Similarity is detected between each of the two current octave intervals and each of the two previous octave intervals. Following point 3 of paragraph 5.2, this
may trigger four possible analogies between intervals. But point 2 is also true: in particular, as the two high notes of both instants are equal (same absolute pitch, same duration), then there is an analogy between these two high notes, idem for the two extreme notes. See scores 1 and 2 of figure 3.

\[
[1a(1a, 1b)] :: [2a(2a, 2b)] \quad (1)
\]
\[
[1a(1a, 1c)] :: [2a(2a, 2c)] \quad (2)
\]

Concerning now the two low intervals, each high notes is included inside each analog intervals of analogy 1. Hence the analogy between these two intervals follows point 1 of paragraph 5.2, that is, a motive is created. See score 3 of figure 3.

\[
[1a(1a, 1b)(1b, 1c)] :: [2a(2a, 2b)(2b, 2c)] \quad (3)
\]

Are also registered the intervals between current and precedent instants, in particular the unisson intervals: between 1a and 2a, between 1b and 2b and between 1c and 2c.

### 7.3 Instant #3

The intervallic similarity between 2a and 3a and between 1a and 2a triggers new analogy. See score 4 of figure 3.

\[
[1a(1a, 2a)] :: [2a(2a, 3a)] \quad (4)
\]

The intervallic similarity between 2a and 2b and between 3a and 3b triggers an analogy.

\[
[2a(2a, 2b)] :: [3a(3a, 3b)] \quad (5)
\]

Similarity between the analog-interval of analogies 1 and 5 leads (see score 5):

\[
[[1a(1a, 1b)][2a(2a, 2b)]] :: [[2a(2a, 2b)][3a(3a, 3b)]] \quad (6)
\]

We will not list all the possible analogies, and will prefer focusing here on important ones. In particular, as for analogy 3:

\[
[2a(2a, 2b)(2b, 2c)] :: [3a(3a, 3b)(3b, 3c)] \quad (7)
\]

Similarity between the analog-interval of analogies 3 and 7 leads (see score 6):

\[
[[1a(1a, 1b)(1b, 1c)][2a(2a, 2b)(2b, 2c)]] :: [[2a(2a, 2b)(2b, 2c)][3a(3a, 3b)(3b, 3c)]] \quad (8)
\]

### 7.4 Instant #4

Similarity of time interval between 2a and 3a and between 3a and 4a, induces an extension of analogy 4 (see score 7):

\[
[1a(1a, 2a)(2a, 3a)] :: [2a(2a, 3a)(3a, 4a)] \quad (9)
\]

Similar as analogies 3 and 7:

\[
[3a(3a, 3b)(3b, 3c)] :: [4a(4a, 4b)(4b, 4c)] \quad (10)
\]

Similarity between the analog-interval of analogies 7 and 10 leads:

\[
[[2a(2a, 2b)(2b, 2c)][3a(3a, 3b)(3b, 3c)]] :: [[4a(4a, 4b)(4b, 4c)(4c, 4d)] \quad (11)
\]

### 7.5 Instant #6

Now relative intervals similarities between the beginning of this motive and the previous one are detected (see score 9).

\[
[[1a, 1b)(1b, 1c)] :: [(6a, 6b)(6b, 6c)] \quad (13)
\]

### 7.6 Instant #7

The repetition is detected (see score 10):

\[
[6a(6a, 6b)] :: [7a(7a, 7b)] \quad (14)
\]

and is compared with the first repetition (see score 11 and 12):

\[
(1a, 2a) :: (6a, 7a) \quad (15)
\]

\[
[[1a(1a, 1b)(1b, 1c)][2a(2a, 2b)(2b, 2c)]] :: [[6a(6a, 6b)(6b, 6c)][7a(7a, 7b)(7b, 7c)] \quad (16)
\]

### 7.7 Instant #8

Idem (see score 13):

\[
[[1a(1a, 1b)(1b, 1c)][2a(2a, 2b)(2b, 2c)]] :: [[6a(6a, 6b)(6b, 6c)][7a(7a, 7b)(7b, 7c)] :: [[8a(8a, 8b)(8b, 8c)]] \quad (17)
\]
7.8 Instant #14

Following similar way, after several sequencings, the new motive is plainly detected (see score 14):

\[
[1a(1a, 2a)(3a, 4a)] \quad \vdash \quad [11(11, 12)(13, 14a)] \quad (18)
\]

\[
[(8a, 9a)] \vdash [14a, 14b] \quad (19)
\]

Moreover, a very interesting relationship of interval is induced by similarity between analog-intervals (see score 15):

\[
[(3a, 4a)(8a, 9a)] \vdash [14a, 14b] \quad (20)
\]

7.9 Instant #18

The three occurrences of main motive (see score 16) is detected in a similar way. And also the repetition of a same chord (see score 17).

7.10 Instant #34

Finally, the equivalence between the first and second half of the whole example is detected (see score 18).

8 Discussion

It can be remarked that we share Meyer’s idea concerning musical listening, of a constant relation of the present instant with known similar context (either experienced in the past of the work or learned as an aspect of musical style), but not the other part – the most important one, according to him – of his theory, namely the expectation of learned continuation. In a phenomenologic terminology, this means that we take into consideration the retentional aspect of perception, but not the protentional one. It would be possible – and also necessary, if we would want to prolonge the cognitive metaphor – to implement the protentional part, but we would like to know if it is possible for a cognitive system such as a computer simulation, to avoid protention. We would tend to think that, when facing with complex environment, protention is necessary, because protentional capacity has some good evolutionist reasons to exist.

Although our system gets inspired by cognitive researches on inductive mechanisms and analogy, its architecture has been established following pragmatic considerations and phenomenologic intuitions. The cognitive modelling is used here only as a kind of metaphor, in a biomimetic demarche. It would be of a greatest interest to build a cognitively founded model, by a collaboration with experimental cognitive psychology and in particular by measuring the parameters of this model through experimental measurements.

\textit{kanthume} is implemented as a library of Ircam musical representation software OpenMusic. The present version (OM\textit{kanthume} O.1) displays the results of its analysis through list of texts, as shown in previous paragraph. Added to the problematic of conception of cognitive modelling, arises then the question of interface and ergonomy. The result of the analysis has to be displayed graphically in a kind of network of relations, above the score itself. Because of its complexity – not graphically representable and in fact not catchy for human – this network should not be entirely displayed, but only a part of it. The user should be able to navigate inside this network, by choosing temporal objects and hierarchical level of the network. Finally, in our first version, following standard algorithmic, the different hypotheses are considered sequentially. In order to follow carefully the cognitive metaphor, we would undoubtedly need to consider a parallel model, for example by implementing a multithreaded version.

Those are the kinds of questions that are considered in my current PhD, directed by Emmanuel Saint-James (LIP6, Paris VI) and Gérard Assayag (Musical Representation Team, Ircam).

\textbf{References}

Bacon, F. (1620). \textit{Novum Organum}.


Hume, D. (1748). \textit{An Enquiry Concerning Human Understanding}.

Kant, I. (1781). \textit{A Critique of Pure Reason}.


Mill, J. (1866). \textit{A System of Logic}.


\footnote{See website for up-to-date developments: \url{http://www.ircam.fr/equipes/repmus/artillot}.}
Figure 1: Two note objects linked to onset and pitch hash-tables.

Figure 2: The intervals between pitch values D and F and between E and G, both worth 3 semitones, are linked to the interval hash-table.

Figure 3: *kanthume* analysis of piano reduction of the nine first bars of Beethoven’s fifth symphony.