Physical modeling of oboe-like instruments: influence of the bore conicity and of the pipeneck after the double reed.

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Keywords: physical modeling, double-reed instruments, conical bore, pipeneck

1 INTRODUCTION

Wind instruments have similar basic principles of functioning: the player blows through a valve (one reed, two reeds, or two lips) into the instrument. The acoustic response of the instrument can be seen as a feedback which influences the valve behavior. The production of a sound corresponds to the auto-oscillation of this system. However, in spite of these common features, each class of instruments has its own particularities.

The present paper deals with the simulation of sound production mechanisms (physical modeling) by oboe-like instruments (such as the oboe, bassoon, ...). Starting from the instrument the *closest* to the oboe, the clarinet (see section 2), two main differences between oboe-like instruments and clarinet-like instruments are analysed, both from an aero-dynamical and an acoustical point of view. These two differences are the shape of the bore, which is conical in the case of the oboe and cylindrical in the case of the clarinet, and the mouthpiece, made of a single reed for the clarinet and of a double reed stapled to a narrow pipe (called a pipeneck) for the oboe. In the following, these two physical differences will be invastigated, in order to understand how they influence the sound production of an oboe. The acoustical effects of the conical bore are well known and have been widely studied in the litterature. However, even if the theory is well established, previous studies have shown that numerical solutions are difficult to be kept stable. In order to cope with this problem, the conical bore is replaced by a quasi-equivalent system, made of two cylindrical pipes, connected perpendicularly to the double reed and to the pipeneck (see section 3).
There is little litterature concerning the effects of the pipeneck, but it seems obvious that its influence on aero-dynamical phenomena should be taken into account. Previous studies [1] have suggested that a non-zero static pressure applies on the reed. This pressure progressively decreases along the pipeneck because of visco-thermal and/or turbulent losses. A simple physical model is proposed. Properties of the nonlinear relation between the volume-flow between the reeds and the pressure difference between the mouth and the begining of the bore are studied and compared to the clarinet case (see section 4.3). Three qualitatively different behaviors of the model have been found to be possible (according to the parameter's), including hysteresis for two of them. Analytical predictions are confirmed by numerical simulations.

2 BASIC MODEL OF SINGLE-REED INSTRUMENTS

In this section, the physics of the clarinet is briefly reviewed. Basic models of single-reed (clarinet-like) instruments have been proposed in [2] and [3] for example. The reed may be modelled as a mass/spring/dashpot oscillator, but due to its high resonance frequency ($\simeq 10^4$Hz) compared to the first resonance frequencies of the instrument, the reed behavior is stiffness-dominated (inertia and damping can be neglected). Assuming that the reed is driven by the pressure difference across the reed, the reed dynamics is modelled by:

$$k_s(z - z_0) = (p_m - p) \tag{1}$$

where z (z_0) is the reed position (at rest), k_s is the reed surface stiffness, p_m and p are the excess of pressure in the mouth and in the mouthpiece. As pointed out by Hirschberg [1], in clarinet-like instruments, the volume flow control by the reed oscillation is due to turbulent dissipation. Actually a jet is formed in the mouthpiece after the separation of the flow at the end of the reed channel. Moreover, since the mouthpiece section is much larger than the reed channel section,

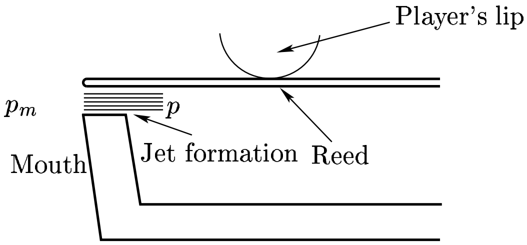

Figure 1: Cut of a clarinet mouthpiece.

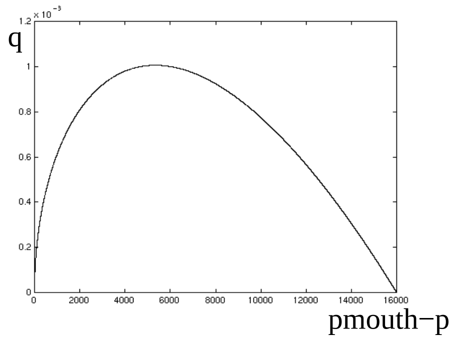

Figure 2: Volume flow calculated according to equation (3) with $\alpha = 1$, $z_0 = 1e^{-3}$m, $l_r = 1.6e^{-2}$m, $ks = 1.6e^7$Pa.m^{-1} and $p_m = 8e^3$Pa.

all the kinetic energy can be assumed to be dissipated by turbulence with no pressure recovery. Then applying the Bernoulli theorem between the mouth and the reed channel leads to:

$$q = \alpha z l_r \sqrt{\frac{2}{\rho_0}(p_m - p)} \tag{2}$$

where q is the volume flow through the reed, l_r is the reed effective width, ρ_0 is the air density. Pressure p is then the acoustic pressure imposed by the response of the bore (assumed linear) to incoming air flow q: $p = g * q$, where g is the impulse response of the instrument (close to the one of a cylindrical pipe). Parameter α is a critical value: it is a semi-empirical *vena-contracta* factor which may take into account a possible contraction of the jet at the input of the reed channel. It can be found throughout the literature $\alpha = 0.6$ according to Wilson and Beavers experiments ([2]) and $\alpha = 1$ (i.e. no *vena contracta*) for Gilbert experiments [4].

Combining equations (1) and (2), the volume flow can be expressed as a well known function of the pressure difference across the reed $(p_m - p)$:

$$q = \alpha l (z_0 - \frac{1}{k_s}(p_m - p))\sqrt{\frac{2}{\rho_0}(p_m - p)} \tag{3}$$

See figure 2 for a graphical illustration of equation (3).

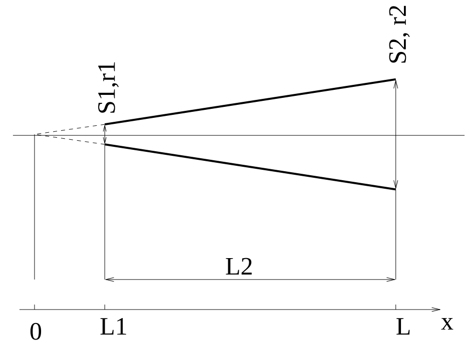

Figure 3: Conical bore

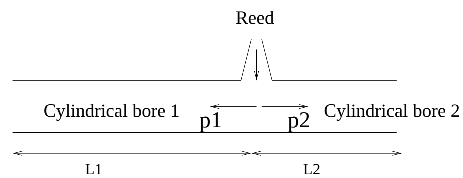

Figure 4: Two cylindrical bores instead a conical.

3 SIMULATION OF THE CONICAL BORE WITH TWO CYLINDRICAL BORES

An oboe bore shape is close to a cone, whereas a clarinet bore can be seen as cylindrical. These bore shapes have different effects on the sound production.
When trying to simulate the equations of the conical bore, a lot of numerical problems have been encountered, especially when trying to keep the oscillations stable.
In order to cope with this problem, a method has been developed, based on the work of Dalmont and Kergomard [5]. The idea is to replace the conical bore with two cylindrical bores of same section perpendicular to the reed, with the length of the first bore equal to the one of the conical bore, and the length of the second bore equal to the one of the truncated part of the conical bore (see figure 4).

3.1 Analytical analysis

To study the response of such a system, the input impedance $Z_{in} = \frac{p_{in}}{q_{in}}$ is studied. This variable provides a complete description of the propagation of the acoustic waves inside the device. This is exactly the same information as contained in the impulse response (linear acoustics hypothesis).
The theoretical frequency of the n^{th} resonance of the input impedance of a conical bore is $f_n = 2n\frac{c}{4L_2}$, with L_2 the length of the cone (see figure 3), while it is $f_n = (2n+1)\frac{c}{4L_2}$ in the case of a cylindrical pipe of length L_2 [6].

At the connecting point, the hypothesis of the

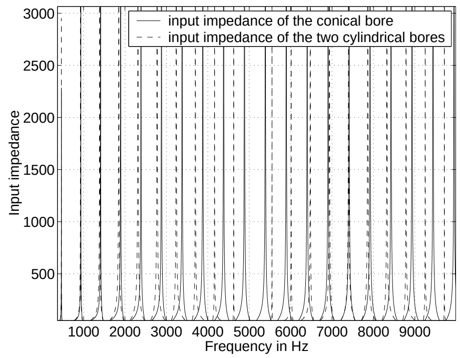

Figure 5: Comparison between the theoretical input impedance of the conical bore and of the two cylindrical bores.

continuity of the pressure and the flow can be written as:

$$p_1 = p_2 = p_{reed} \tag{4}$$

$$q_1 + q_2 = q_{reed} \tag{5}$$

Assuming that the waves are plane in a cylindrical bore, and spherical in a conical bore, these two equations lead to, neglecting the losses due to the internal frictions and to the external radiation:

$$Z_{in} = \frac{\rho_0 c}{S} \frac{1}{\frac{1}{tan(kL_1)} + \frac{1}{tan(kL_2)}} \tag{6}$$

with $k = \frac{2\pi f}{c}$ the wave number, S the bore section.

3.2 Discussion

The theoretical input impedance of a conical bore is:

$$Z_{in} = \frac{\rho_0 c}{S} \frac{1}{\frac{1}{kL_1} + \frac{1}{tan(kL_2)}} \tag{7}$$

The error made is the substitution of $tan(kL_1)$ by kL_1. This error is always small, except for values of kL_1 around the odd multiples of $\frac{\pi}{2}$ (i.e. the value of f close to the odd multiples of $\frac{c}{2L_1}$, which is ten times the theoretical fundamental resonance frequency of the conical bore, see [7], if $L_2 = 10 * L_1$).
It can be noticed in figure 5 two main differences: first of all, there is a huge attenuation of the resonances of the two cylindrical bores around the n^{th} resonance, compared to those of the input impedance of conical bore, with n the multiples of the entire part of $\frac{L_1}{L_1+L_2}$ and $\frac{L_2}{L_1+L_2}$ (in the case of figure 5, this resonance is missing). The other difference is that the resonance frequencies are no longer $f_n = 2n\frac{c}{4L_2}$ but $f_n = 2n\frac{c}{4(L_1+L_2)}$. Fortunately L_1 is always small compared to L_2 [8].
Analytical studies (see [7] and [6]) have shown that taking into account the internal losses and the reflection at the bell's end has the effect of low-pass filtering. Then, the two input impedances are close, and the behavior of the waves inside the device can be supposed

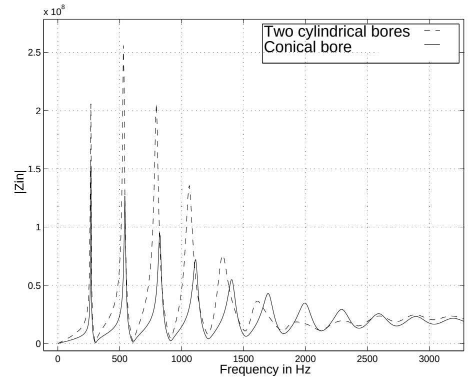

Figure 6: Comparaison between the theoretical input impedance of the conical bore and of the two cylindrical bores, taking into account the losses.

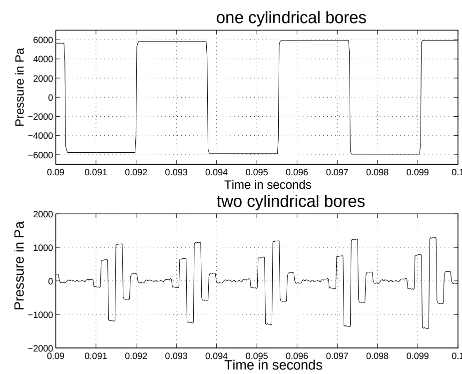

Figure 7: Waveforms obtained when simulating a single pipe connected to a single reed (upper part), and two pipes a single reed (lower part)

to be a good approximation of the one of the conical bore (see figure 6).

3.3 Numerical simulation

In order to test the stability of our method and to judge the influence of the bore conicity on the sound emitted, the sound of two cylindrical bores put on a single reed (i.e. saxophone-like instrument) have been simulated.
The method used was based on *digital waveguide modeling* techniques [9]. A simple model was used, the two bores being modeled by two delay lines, without any filtering modeling the losses. The reed is viewed as an exciting non linear mechanism providing energy to the system. At each step of the algorithm, the pressure and the flow are computed according to the previous steps.
The figures 7 and 8 show the results of two simulations: the upper part of both figures are the waveforms and the *spectra* of a single cylindrical tube ($L_1 = 30cm$) connected to a single reed. The lower part is the simulation of two pipes ($L_1 = 30cm$, $L_2 = 3cm$) connected to a single reed.
These simulations show resonance frequencies con-

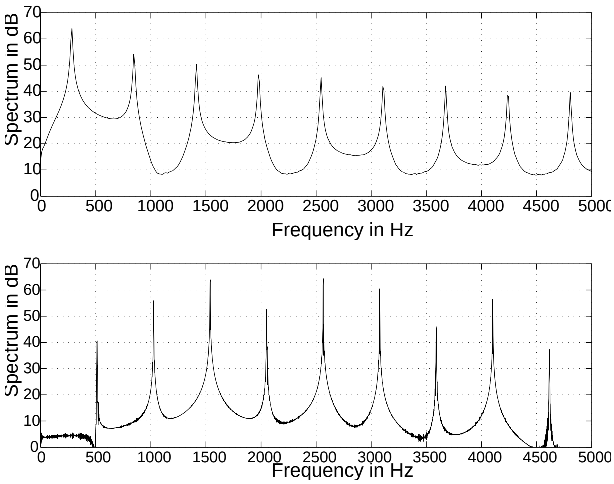

Figure 8: Spectra of the waveforms obtained when simulating a single pipe connected on a single reed (upper part), ans two pipes a single reed (lower part)

sistent with theory (the theoretical fundamental resonance frequency is 515 Hz) for the *spectrum* of the sound, which means that this method is worthwhile for simulating the behavior of a conical bore. The waveform is quite different from the one obtained for a cylindrical pipe, and so is the sound produced. However, without any losses simulated, the sound remains "metallic" and unrealistic.

4 EFFECT OF A PIPENECK DOWNSTREAM OF THE REED

The most obvious difference between single and double reed instruments is probably the number of reeds. However, since in double reed instruments both reeds are the same and have a symmetrical behavior, a single mass model is commonly used [10], like in single reed instruments models. Another difference is that double reeds are stapled to a narrow pipe (a pipeneck). This puts the flow model described in section 2 questionnable.

4.1 Simple model for the pipeneck

In fact, as seen in section 2 in the case of a clarinet, the air jet formed at the reed is supposed to dissipate all its kinetic energy by turbulence in the mouthpiece with no pressure recovery. This hypothesis stands because the bore entrance is much larger than the slit made by the reed opening. On the contrary, in the case of an oboe, the diameter of the pipeneck progressively increases between the reed slit and the input of the bore. The previous hypothesis cannot be applied.

In their article [1], Wijnand and Hirschberg explain that placing a pipeneck between the mouthpiece and the bore of the instrument probably causes an additional pressure drop between the mouthpiece pressure p and the pressure at the input of the pipe p_p. In a quasi-stationary flow model, this additional pressure drop $(p - p_p)$ can be expressed by a discharge-losses-coefficient C_d. We propose to express $p - p_p$ as a function of the air velocity at the input of the neck:

$$p - p_p = \frac{1}{2}\rho_0 C_d \frac{q^2}{S_n^2} \tag{8}$$

where S_n is the cross section of the neck and C_d takes into account visco-thermal and/or turbulent losses.

Relations (1), (2) and (3) are still valid, and we are now focusing our attention on the relation $q = \mathcal{F}(p_m - p_p)$. In fact, the instrument is now supposed to impose the pressure p_p. According to equation (3), it is first necessary to find $p_m - p = \mathcal{G}(p_m - p_p)$. After combining equations (1), (2) and (8), it is easily found that $(p_m - p)$ is solution of:

$$\begin{aligned} (p_m - p)^3 - 2z_0 k_s (p_m - p)^2 \quad + \quad ... \\ +k_s^2(z_0^2 + D)(p_m - p) - k_s^2 D (p_m - p_p) \quad = 0 \end{aligned} \tag{9}$$

with $D \triangleq \frac{S_n^2}{\alpha^2 l^2 C_d}$

Obviously, it is possible to solve numerically equation (9). This will be done in section 4.3. However, it is far more interesting to obtain properties of the solution according to to the model's parameter values.

4.2 Analytical results

The following results have been obtained analytically and will be published in a forthcoming paper. For lack of space, demonstrations are omitted here.

According to the model's parameter values, three qualitatively different behaviors have been found:

Type 1: when $0 \leq C_d < \dfrac{3S_n^2}{\alpha^2 l^2 z_0^2}$,
$(p_m - p)$ is a monotonous increasing function of $(p_m - p_p)$. The behaviour is qualitatively similar to the one of the clarinet (coresponding to $C_d = 0$)

Type 2: when $\frac{3S_n^2}{\alpha^2 l^2 z_0^2} \leq C_d < \frac{4S_n^2}{\alpha^2 l^2 z_0^2}$,
$(p_m - p)$ is a multi-valued function of $(p_m - p_p)$ on a certain range of $(p_m - p_p)$. This is responsible for hysteresis (the presence of hysteretic behavior has been highlighted by Mahu ([11]) using numerical simulations, but without specifying necessary conditions on C_d). Moreover, the reed displacement has two possible discontinuities and the reed always jumps between two-opened reed positions.

Type 3: when $\frac{4S_n^2}{\alpha^2 l^2 z_0^2} \leq C_d$,
conclusions are the same as for *type 2*, excepted that the reed jumps from an opened-reed to a closed-reed position or from a small opened-reed to a larger open-reed position. These behaviors are illustrated in section 4.3.

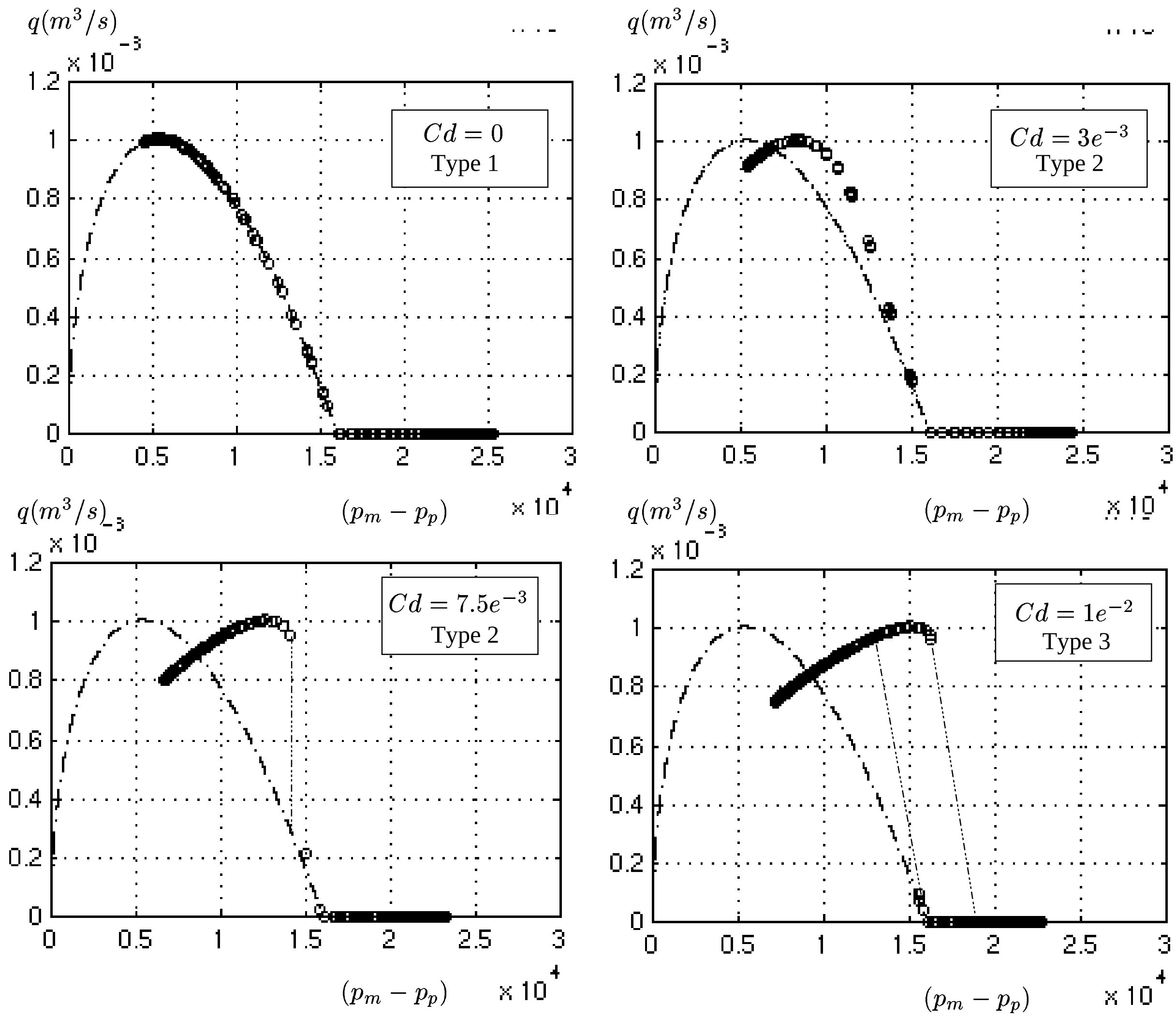

Figure 9: Comparison between the analytical formula for q without pipeneck (eq. 3) (dash-dotted line) and simulation results (noted $\circ$) for different values of the discharge losses coefficient C_d.

4.3 Numerical simulations

Numerical values for parameters are those of figure 2 (except $p_m = 1.6e^4$Pa), plus $Sn = 7.85e^{-7}$m^2. Since we are interested in studying the effect of the pipeneck, we only consider a cylindrical tube for the instrument (length $l_t = 0.72$m). This tube is characterized by a simple reflexion function (a delay plus a low-pass filter).

Equations (1), (2), (9) and the acoustical coupling are solved through an iterative process. According to the analytical results (section 4.2), transitions between *type 1*, *type 2*, and *type 3* occur respectively for $C_d = 7.2e^{-3}$ and $C_d = 9.6e^{-3}$. In figure 9, three simulations are performed for $C_d = 3e^{-3}$, $C_d = 8e^{-3}$ and $C_d = 1e^{-2}$. They confirm that three types of qualitatively different behaviors are possible. On panel *type 2*, it is shown that the reed may jump between two opened-reed positions, whereas in panel *type 3*, the reed jumps from an opened-reed position to a closed-reed position when it closes.

5 CONCLUSION

In this paper two major differences between clarinet-like instruments and oboe-like instruments have been studied: a conical shape and the presence of a pipeneck downstream of the double reed.

First of all, the well known effects of the conical bore have been investigated. Whereas the input impedance of a cylindrical bore shows resonance at all the odd multiples of the fundamental resonance frequency, the input impedance of a conical bore has its maxima at the even multiples of the fundamental (which means all the multiples of another fundamental frequency two times larger than for a cylindrical pipe). A method which simulates this behavior has been studied. It consists in simulating two cylindrical bores connected perpendicular to the reed. Numerical simulation have shown the consistency of the algorithm with the theory

Concerning the effect of the pipeneck, a model has been proposed, leading to a nonlinear relation between air pressure close to the reed and air pressure at the begining of the bore (i.e. end of the pipeneck). This relation is parametered by the discharge losses coefficient associated with the pipeneck. Three qualitatively different behaviors have been highlighted, two of them being hysteresic. Analytical conditions to obtain each of the three different behaviors are given in the present paper.

Experiments are being carried out to check the validity of the model, and the magnitude order of C_d in oboe-like instruments. Indeed, nothing guarantees that *type 2* and *type 3* behaviors will be observed. However, preliminary stroboscopic visualisations of double-reed oscillations (made by R. Caussé) confirm that the reed may "jump" during its cycle between two positions. The present model will be used in sound synthesis applications. Sound examples will be presented at the conference.

6 REFERENCES

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