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Dubium sapientiae initium. (R. Descartes)

Different resolutions for different informations

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INTRODUCTION

The existing algorithms for **analysis, synthesis and transformation of sounds** ask the users to have a high competence on signal processing. To make the access to these efficient algorithms easier, it is desirable to **let the algorithms adapt themselves** to the signal and to the transformation needed.

One of the key parameter for nearly all the algorithms which analyze or process the sounds by using time-frequency representations or sinusoidal models is **the size of the analysis window**: here we show some examples about how this parameter is linked to the resolution of a signal analysis, and which informations can be highlighted choosing a certain size instead of another.

We are at present studying and developing algorithms for the **automatic adaptation of the analysis window size**. The first method approached is the parallel evaluation of multiple choices of the parameters and the subsequent combination of the final outcome through a comparison between their features.

ANALYSIS-SYNTHESIS SCHEME

An **audio signal** can be represented as a function

$$s : \mathbb{R} \to \mathbb{C},\ s \in L^2(\mathbb{R})\ .$$

A system for the analysis and synthesis of a signal is composed of

- a set of functions called **time-frequency atoms**
$$\{\phi_\gamma\}_{\gamma\in\Gamma} \subset L^2(\mathbb{R}) \tag{1}$$
- a **time-frequency transform** operator associated to the above set
$$T_s(\gamma) = \int s(t)\overline{\phi_\gamma(t)}\mathrm{d}t = \langle s, \phi_\gamma\rangle \tag{2}$$
- a **reconstruction formula**
$$s(t) = \int_{\gamma\in\Gamma} T_s(\gamma)\phi_\gamma(t)\mathrm{d}\gamma \tag{3}$$

FRAMES THEORY

For the reconstruction formula to exist, the set $\{\phi_\gamma\}_{\gamma\in\Gamma}$ has to satisfy a condition obtained by relaxing the definition of an orthonormal system in a Hilbert space.

Definition 1 *The set $\{\phi_n\}_{n\in\Gamma}$ is a frame of $L^2(\mathbb{R})$ if A and B exist such that they are positive and for every $s \in L^2(\mathbb{R})$*

$$A\|s\|^2 \le \sum_{n\in\Gamma} |\langle s, \phi_n\rangle|^2 \le B\|s\|^2\ , \tag{4}$$

where thanks to the features of the Hilbert space $L^2(\mathbb{R})$ we can suppose Γ to be countable.

WINDOW FUNCTIONS

We consider a **window function** as a function $h \in L^2(\mathbb{R})$ such that $\|h\| = 1$ and $h(t) = h(-t)$;
modifying h with a time translation of a factor t_0 and a frequency scaling by a factor ν_0 we obtain the set $\{h_{t_0,\nu_0}\}_{(t_0,\nu_0)\in\mathbb{R}^2}$ where

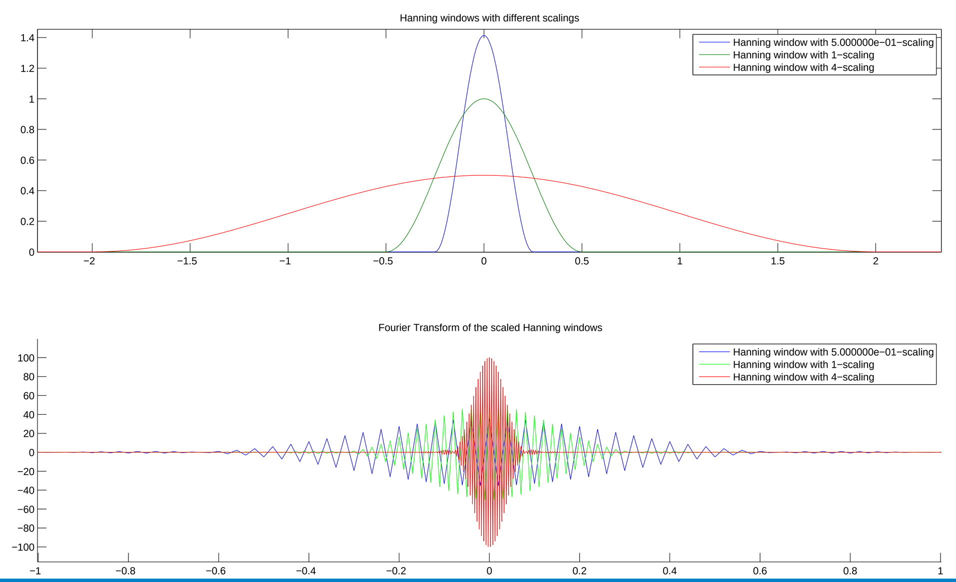
$$h_{t_0,\nu_0}(t) = h(t - t_0)e^{2\pi i\nu_0 t}\ . \tag{5}$$

Remark 1

- The **Short Time Fourier Transform** is the time-frequency transform associated to the set $\{h_{t_0,\nu_0}\}_{(t_0,\nu_0)\in\mathbb{R}^2} \subset L^2(\mathbb{R})$
- Such set contains infinite countable frames of window functions for $L^2(\mathbb{R})$

A frame can be built by conveniently combining different sets of atoms derived from different window functions.
We are particularly interested to the case of the scaling of a same window function, which modifies its **time-frequency spread**,

$$h_l(t) = \frac{1}{\sqrt{l}}h\left(\frac{t}{l}\right)\ . \tag{6}$$



ADAPTIVE SPECTROGRAM

The **Spectrogram** of a signal s is the squared modulus of the STFT of the signal

$$PS_s(t_0, \nu_0) = \left|\int s(t)h(t - t_0)e^{-2i\pi\nu_0 t}\mathrm{d}t\right|^2\ . \tag{7}$$

It is always positive and with certain exceptions it can be treated as a probability distribution.

It is possible to perfectly represent the signal s in infinite many different ways, by choosing an appropriate set of multi-index $\Gamma \subset \mathbb{R}^2$ so that the relative frame $\{h_{t_0,\nu_0}\}_{(t_0,\nu_0)\in\Gamma}$ is dense enough in the time-frequency plane.

Given PS_s^h the spectrogram of a signal s through the frame built on the window h, a **sparsity measure** can be defined to evaluate the energy concentration in the analysis PS_s^h.

The notion of **best energy concentration** is not uniquely defined and is linked to the desired trade-off between time and frequency resolution.

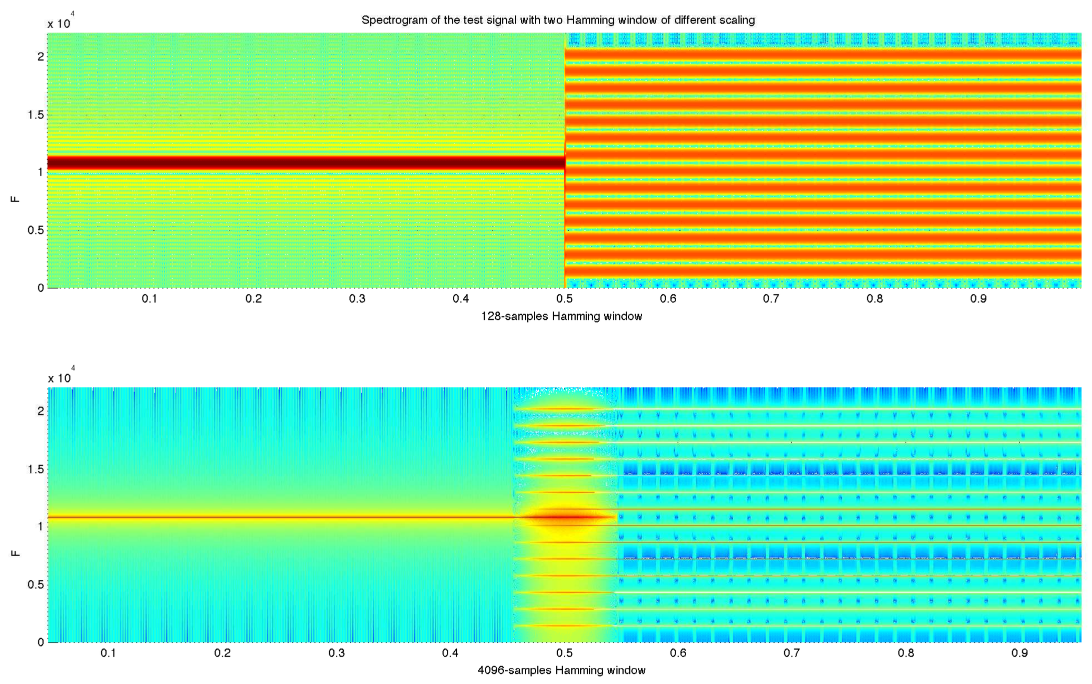


Fig.2 Spectrogram of a test signal with two Hamming window of different scaling.

As shown in [BF01] it is possible to extend the notion of **entropy** to certain time-frequency representations, such as the spectrogram, to define on them a class of sparsity measures.

Definition 2 *Given a time-frequency representation $C_s \in L^2(\mathbb{R}^2)$ of the signal s, its* Rényi entropy *of order $\alpha > 0, \alpha \ne 1$ is*

$$H_\alpha(C_s) = \frac{1}{1-\alpha}\log_2 \iint C_S^\alpha(t,\nu)\mathrm{d}t\mathrm{d}\nu \tag{8}$$

Given a signal s and n different spectrograms $PS_s^{h_1}, ..., PS_s^{h_n}$ of the same signal based on different windows $h_1, ..., h_n$ we choose as the **best spectrogram** the one which minimizes the Rényi entropy measure,

$$l^* = \arg\min_{l=1,...,n} H(PS_s^{h_l})\ . \tag{9}$$

ALGORITHM AND EXAMPLES

- Input and main parameters: the signal s, the set of window functions $h_1, ..., h_n$, the Rényi entropy order α
- Computation of the n different spectrograms
- Evaluation of the Rényi entropy values on the global analysis
- Choice of the best spectrogram

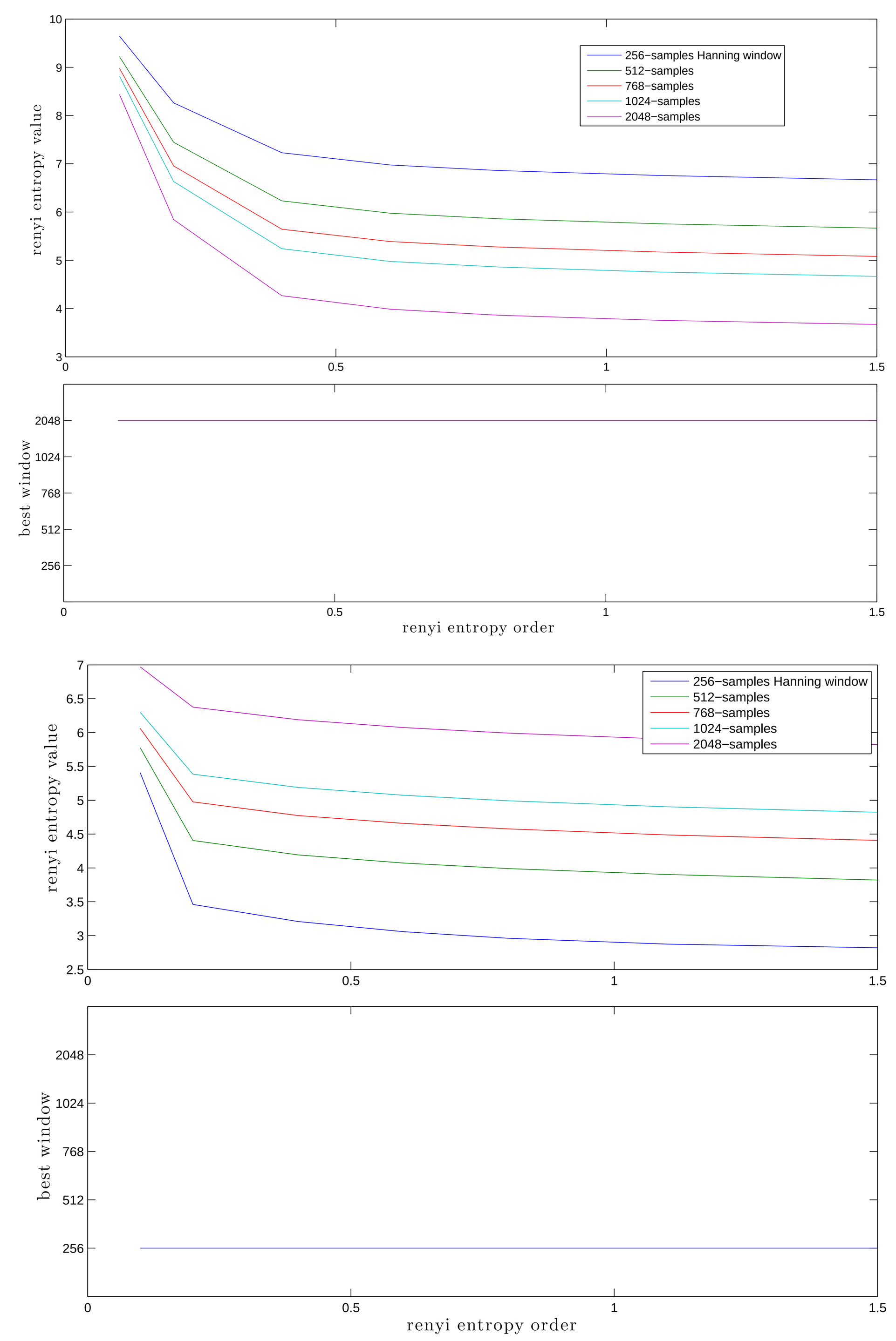


Fig.3 The choice made by the algorithm for two basic signals: two different sinusoids in the upper figure, an impulse in the lower. In these cases, the choice is α-independent

RESEARCH PROGRAM

- Development and investigation on the best strategy to set the hop size of the different spectrograms, in order to enhance the compatibility of the different analysis
- Definition of a strategy for time and frequency localization of the Rényi entropy evaluation, to obtain a local adaptation of the window size to the signal
- Investigation on the relation between the Rényi entropy order α and the choice of the best spectrogram by the algorithm

REFERENCES

- [BF01] R.G. Baraniuk, P. Flandrin, A.J.E.M. Janssen, O.J.J. Michel, *Measuring Time-Frequency Informaion Content Using the Rényi Entropies*; IEEE Transaction on Information Theory, vol.47, n.4, 2001.
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