Effort-based analysis of bowing movements: evidence of anticipation effects

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Abstract

Anticipatory behaviours are known to occur in music performance, notably on the control movements of instruments such as piano or drums. We studied such effects on bowed string movements, corresponding to a case where the control on sound is continuous. Movements were measured with an optical motion capture system combined with sensors on the bow. Bowing movements were analysed and compared on the basis of underlying effort costs, determined from their velocity profiles. Precisely, we used movement models that assume that jerk or impulse are minimized. These models were synthesized based on measurement data and then compared to velocity and acceleration profiles. Results on various musical cases involving separate strokes, scales, mixed bowing techniques and rhythms showed that this methodology can account, to some extent, for the different effort strategies used by the players. The presented modelling provides evidence of anticipatory behaviour during bowing movements.

1 Introduction

As evidenced by the extensive development of gesture-based interfaces for music and the increasing number of conference sessions dedicated to gesture controlled music, an important trend in computer music lies in the possibilities of controlling computer-generated sounds in real-time through the use of movements and physical gesture. This can enable electronic musicians to involve their body and motion in performance, recreating essential

interactions found between musicians and their instruments (Leman, 2007). In that perspective, the study of different control strategies and constraints that acoustic instrument players must manage can bring important insights for such new approaches in electronic music performance.

A common assumption in music performance research is to consider music playing as a sequence of actions. Particular studies showed that in such motion sequences, each element can be affected by its neighbours. Pianists, for example, can modify their finger movements to anticipate the subsequent keystrokes one or two keystrokes ahead (Engel et al., 1997). Such an anticipatory movement behaviour can supposedly ease timing constraints inherent to music performance, e.g. tone production or rhythm, similarly to coarticulation phenomena found between phones in speech production (Engel et al., 1997; Godøy, 2004; Ortmann, 1929). From the analysis of drummers' playing, Dahl (2000) found that the performance of mixed stroke types influence stroke movements and timing. For example, in the case of patterns composed of one accented and three unaccented strokes, drummers typically prepare the accented stroke by raising their drumstick higher at the end of last unaccented stroke. They also lengthen the accent interval, which as a result gives more emphasis to the accent. These changes exemplify two underlying mechanisms that participate in anticipation, namely cognitive and biomechanical constraints. Analyses of piano player movements indeed revealed that cognitive chunking processes affect finger timings while biomechanical constraints have a notable effect on finger motion trajectories (Loehr and Palmer, 2007). Such anticipatory behaviours are in our view symptomatic of the different types of constraints (i.e. biomechanical, acoustical, cognitive, aesthetic) that acoustic instrument players must face to create expressive music.

The goal of this paper is to investigate anticipatory behaviours occurring with self-sustained instruments, like bowed strings or winds, where players have a continuous control on sound. On bowed strings for example, the production of sound results from the friction of bow hairs on the strings, moving at a certain velocity (Cremer, 1984). This acoustical mechanism enables bowed string players to continuously master their sound through a precise control of the bow. From this continuous control, they actually achieve different expressive cues as shown in bowing technique studies (Rasamimanana et al., 2006) or in analysing different performance versions (De Poli et al., 1998; Winold et al., 1994). In this perspective, it is interesting to study how possible anticipatory behaviours may happen in such instrument and investigate how anticipation challenges the instrument sound control movements. Previous studies showed some anticipation effects in violin playing, between

left and right hands (Baader et al., 2005; Wiesendanger et al., 2006). While these studies examined coordination issues between fingering and bowing, we here aim to focus on anticipation occurring within bowing movements.

To carry out this study, we consider bowing movements as skilled movements, i.e. movements developed through training and practice to achieve certain objectives associated with a task. Under this assumption, bowing movements can be analysed under the scope of performance constraints optimization, and we can hence relate different effort costs to their execution. Nelson (1983) introduced and described elementary principles underlying skilled movements. He defined different objectives related to physical economy of effort, e.g. minimizing energy cost or time cost, and showed that these objectives underlie different classes of skilled movements with specific velocity patterns. With this method, Nelson (1983) modeled and evaluated aspects of motor control strategies in jaw movements during speech. He also briefly reviewed violin bowing movements and pointed to possible effort costs. Links between underlying organizing principles and shapes of velocity patterns were also established by Hogan (1984) in the case of voluntary arm movements. Hogan (1984) proposed a mathematical model assuming smoothest possible movements to predict the velocity profiles of such movements. Inversely, it was also shown that the shape of velocity profiles can be related to effort costs as reported in (Ostry et al., 1987; Perkell et al., 2002) for speech movements.

We hence propose in this paper to compare different bowing movements on the basis of underlying effort costs. It was previously found that the execution of different bowing techniques, e.g. *Détaché* (sustained strokes) and *Martelé* (sharp almost percussive strokes), implies specific bow velocity profiles with characteristic bow accelerations at the start and end of strokes (Rasamimanana et al., 2006). Besides, different bow velocity patterns were found depending on tempo (Rasamimanana et al., 2007). These characteristics could interestingly be related to different efforts underlying bowing movements. To carry out this study, we analysed bowing movements using an analysis/synthesis method based on the kinematic formalism presented in (Nelson, 1983). First, this method allows us to study, quantify and interpret bowing variations due to adjacent strokes. Second, this approach also allows for the description of the bowing techniques reported in (Rasamimanana et al., 2006, 2007) with a different point of view.

This article starts with a brief presentation of the kinematic formalism and its adaptation to the study of bowed string movements. The measurements of real players' movements are then described. The analysis of measurements and the results are finally given and discussed.

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2 Kinematic description of bowing movements

Instrument players achieve different *objectives*: on a first level, they aim at fulfilling objectives related to *music*, from technical aspects such as rhythm or tone production to expression. However, it was evidenced that on a second level, they also try to satisfy performance objectives that could be related to *physical economy* (Nelson, 1983). These musical and physical objectives define a set of constraints that influences the shapes of bow strokes velocity profiles. This section presents the kinematic formalism we used to describe bowing movements.

2.1 Formulation of physical constraints for bow control

We approximate bowing movement by a linear displacement of a mass m, along a dimension x. The mass spans the distance D in the movement time T, starts and ends with a null instantaneous velocity v. Moreover, an external force $F(t)$ is applied on mass m: the amplitude of this force is assumed to be bounded by a limit F_{max}. We consider $u(t) = F(t)/m$, generally referred as *control action* (Nelson, 1983), homogeneous to an acceleration and bounded by $U = F_{max}/m$. Considering a dissipative friction term $f_d(t)$, the system of equations describing the mass movement can be formalized as:

$$\begin{cases} \dot{x}(t) = v(t) \\ \dot{v}(t) = u(t) - f_d(t) \end{cases} \tag{1}$$

with the following conditions

$$\begin{cases} x(0) = 0, & x(T) = D, \\ v(0) = 0, & v(T) = 0, \\ |u(t)| \leq U \end{cases}$$

In this paper, we use this kinematic description for the longitudinal movements of the bow with respect to the instrument (violin, viola or cello). Adopting this formalism for each bow stroke, we obtain that strokes start and end with null velocity but not necessarily null acceleration (corresponding to bow changes), strokes have a given bow length D and duration T. The control action $u(t)$ actually corresponds to the acceleration that players give to the bow.

Moreover, one can notice that system (1) formulation is particularly straightforward. It should be stressed that similarly to (Nelson, 1983), the aim here is not to give a detailed and complete model of bowing movements,

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but rather to propose a simple and efficient description to evidence the relationships between physical dynamics, physical constraints, and performance objectives.

Furthermore, we assume an ideal movement without dissipative forces, i.e. $f_d(t) = 0$. This hypothesis is of course debatable in the case of movements related to bowed strings, since the vibration of the string is made possible from the friction force applied by the bow (Cremer, 1984). Nevertheless, on a first approximation, the friction characteristic can be modelled as a viscous friction (Serafin, 2004). This case is actually addressed by Nelson (1983, see appendices): although some changes actually appear on the analytical solutions of (1), especially on the absolute values of velocities, the principal characteristics of the velocity profiles remain unchanged. As a consequence, the relationships between physical aspects and performance objectives remain essentially the same. For this reason and for the sake of simplicity, dissipative forces are neglected as a first approach.

The model presented in system (1) is henceforth used to describe the kinematics of string bowing movements. We now introduce the concept of performance objectives and their consequence on the solutions of system (1).

2.2 General definition of performance objectives and impact on velocity profiles

To solve the equation system (1), a function has to span the distance D in the time T. There is actually an infinity of functions that may satisfy such conditions. However, as stated in (Nelson, 1983), solutions corresponding to skilled movements should also satisfy performance objectives that can be expressed as the minimization of a physical "cost" associated with the movement. This eventually defines supplementary global constraints. In this paper, we focus on the subclass of *unimodal* solutions, i.e. with velocity profiles whose slope sign changes exactly once: such movements indeed represent a certain efficiency of movement, since they correspond to a single accelerative phase and a single decelerative phase. Besides, such movements are also consistent with previously studied bow strokes (Rasamimanana et al., 2006). The next paragraph presents different performance objectives for such solutions.

Performance objectives are defined as minimizations of physical cost measures. They typically relate to *time*, *force*, *impulse*, *energy*, *jerk* (i.e. acceleration variations) and can be expressed analytically, as presented in appendix A. These costs actually define specific velocity profiles as shown on Figure 1 where different solutions are plotted for a fixed set of measured

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data: $D = 0.63m$, $T = 1s$ and $U = stroke\_max\_acceleration = 25m/s^2$.
We can see that:

- Minimizing force (A), yields to a triangular pattern for instantaneous velocity, with slopes $(A_m, -A_m)$,

- The minimum impulse solution (I) has a trapezoidal shaped pattern with start and end slopes equal to U and $-U$ and minimum peak velocity V_m,

- The minimizations of energy (E) and jerk (J) costs yield to velocity patterns with smooth bell shapes.

The analytical solutions satisfying (1) and the above performance objectives are given in appendix B.
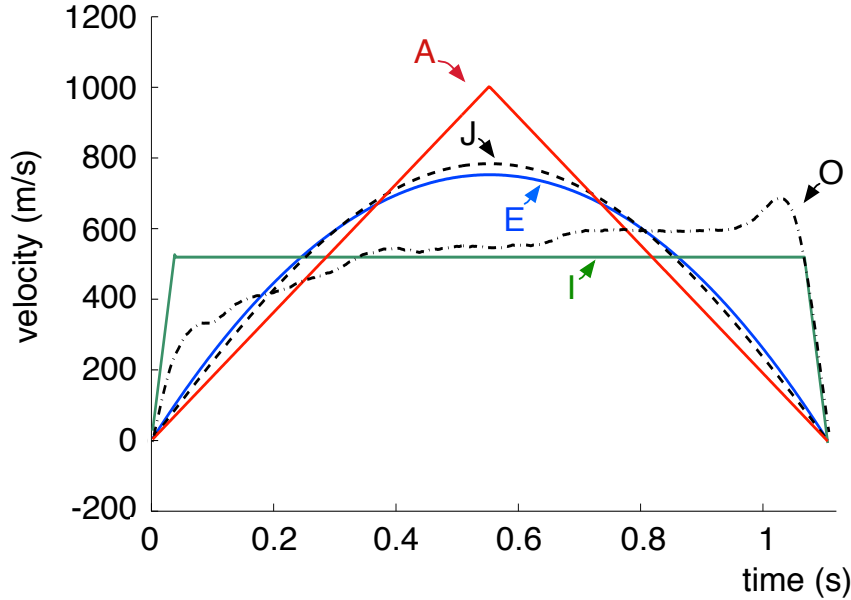
Figure 1: Velocity patterns minimizing different cost objectives. (O): original measured velocity pattern. (I): minimum impulse. (E): minimum energy. (J): discrete minimum jerk. (A): minimum force. Adapted from (Nelson, 1983)

Minimum impulse and minimum jerk are cost objectives particularly relevant to bowing movements considered in our study and are further detailed

below.

2.3   Minimum impulse and minimum jerk objectives

Minimum impulse solutions minimize the total impulse, i.e. the time integral of the control action function u, over the stroke, given D, T, and U. However, we can also consider these solutions as minimizing the velocity variations over the stroke given D, T and U. Solving system (1) under this constraint indeed yields a trapezoidal velocity profile where the velocity variations are $+U$, 0, and $-U$: this form has often been reported as "*Bang-Zero-Bang*". In this perspective, these solutions can be particularly appropriate to describe bowings: players can keep a relatively constant bow velocity to obtain a sustained sound.

Minimum jerk movements, which minimize acceleration transients, are widely used in motor control studies to describe free human movements. Two types of minimum jerk solutions can be defined according to the considered movements, i.e. discrete or cyclical (Nelson, 1983; Hogan and Sternad, 2007). Discrete movements are characterized with well defined start and stop phase, and are separated by a pause. Thus, discrete minimum jerk solutions specifically impose null velocity and null acceleration at the beginning and end of each movement. Cyclical movements correspond to sequences of repeating patterns with no pause. In this case, cyclical minimum jerk solutions have null velocity and non-null acceleration at the beginning and end of each movement. Moreover, as we consider repeated "back and forth" movements, acceleration is maximum at those moments. These two types of solutions have distinct velocity patterns, both characterized by a bell shape. These solutions are relevant for bowing in the case of stopped or repeated strokes.

Using measured times T, bow lengths D, bow accelerations u from real players data and the analytical solutions to system (1), different bow velocity patterns can be synthesized satisfying different performance objectives. It is then possible to assess performance objectives involved in the playing by comparing synthesized and measured velocity profiles.

3   Setup and procedure

This section describes the setup, protocole and musical material used for the study.

3.1   Sound and movement measurements

We used a Vicon System 460 optical motion capture system to measure bow motion. Six M2 cameras were placed around the instrumentalist, providing a spatial resolution below 1mm at a frame rate of 500Hz, on a volume of approximately $1m^3$. Six markers were placed on the instrument, four on the violin table, one on the nutmeg and one on the tailpiece to indicate the strings position. Three markers were placed on the bow. Figure 2 shows the marker placement.

An additional 3 axis ADXL202 accelerometer was fixed at the frog of the bow. Accelerometer data was digitized at 500Hz and transmitted to a laptop for recording. To guarantee post-recording synchronization between motion capture data and accelerometer data, the sound track was recorded simultaneously by each sensing system. The coupling of motion capture data and accelerometer data grants a precise time and space measurement of players position, velocity and acceleration (Rasamimanana, 2008).

The markers and sensors placed on the bow added less than two grams, mainly at the frog (the bow was 62g and the frog was 17g): the players did not feel being disturbed by the system.

3.2   Procedure

We asked three bowed string players to perform five different musical situations. All were advanced level players with eight to ten years of practice. Five tasks were asked:

- $Task1$: the violinists played series of ten isolated strokes, i.e. quarter notes interleaved with pauses, in *Détaché* and in *Martelé* (see Figure 5 for music score).

- $Task2$: the violinists played one ascending and descending scale (one octave), in *Détaché* and in *Martelé* (see Figure 5 for music score).

- $Task3$: the violinists played an exercise mixing *Détaché* and *Martelé* bowings. It consisted in a pattern of four quarter notes where the first two notes are *Détaché* and the last two notes are *Martelé*. The reverse pattern, first two notes *Martelé* and last two note *Détaché*, was also recorded. Each pattern was repeated six times, therefore constituting a set of 36 patterns mixing the two bowings (see Figure 6 for music score).
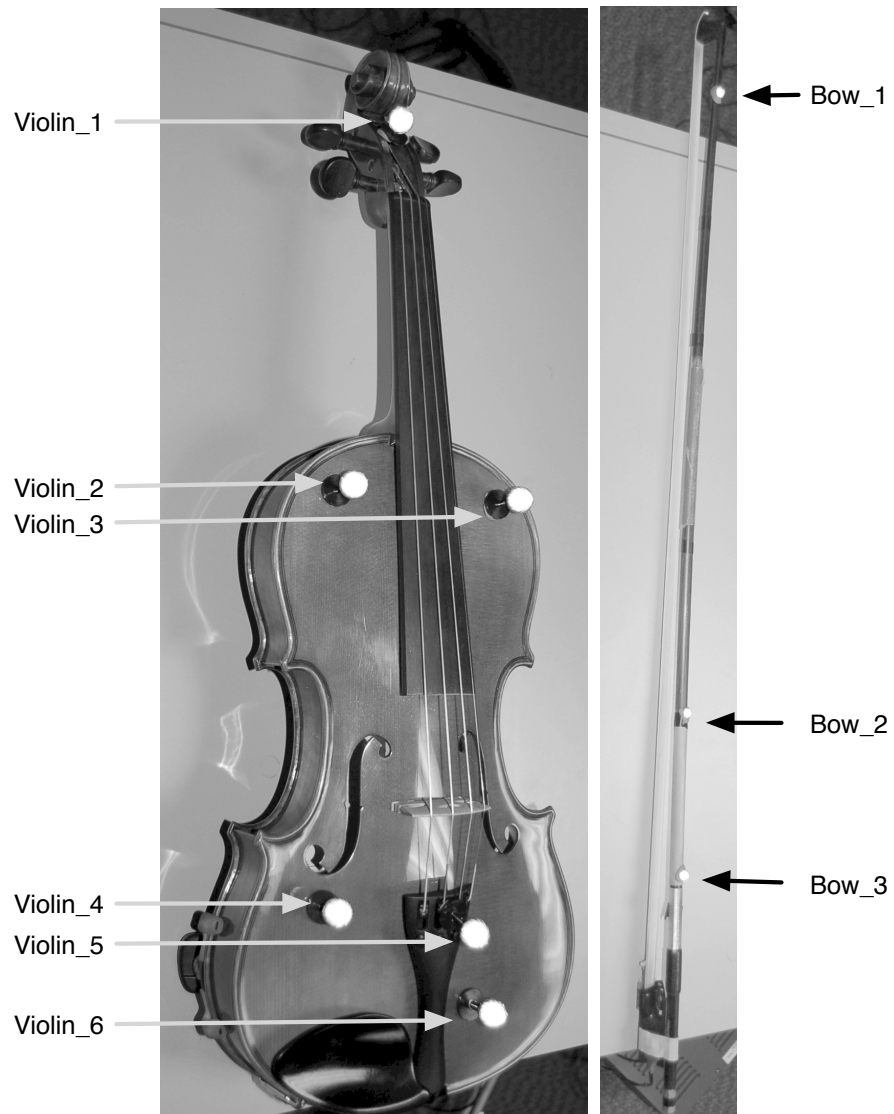
Figure 2: Marker placement to measure violin and bow movements through motion capture.

- *Task*4: the violinists played an exercise mixing different rhythms. The rhythmic pattern was constituted of a quarter and four sixteenth notes, repeated four times on different notes (see Figure 7 for music score).

- *Task*5: the violinists performed a *Détaché accelerando* from moderate tempo (80 bpm) to "as fast as possible", tied with a *decelerando* (see Figure 8 for music score). This exercise was the same as reported in (Rasamimanana et al., 2007).

No specific indications were given for bowing directions: violinists alternated downbows and upbows. For the tasks 1 to 4, the players were asked to play at a moderate tempo, 80 bpm and at a *forte* dynamic.

It should further be noticed that we did not aim here to perform statistics on a large number of players, as a great variability would be expected (Winold et al., 1994; Dahl, 2000; Rasamimanana, 2008). We rather focused on an in depth study of a smaller number of expert instrumentalists, taking into account each idiosyncratic playing.

4 Method

This section details the analysis/synthesis process we develop in this study. First we present the synthesis method for cost-based velocity profiles based on measured data features. Second, we present the method of computing differences between measured and synthesized profiles, which permits quantitative assessment of underlying effort costs.

4.1 Computation of bowing parameters

The first step corresponds to extracting bowing parameters from measured data. The markers on the violin are used to define a frame of reference. The transverse displacement of the bow, $x(t)$, is then calculated in this frame from the projection of bow marker positions. Motion derivatives of the bow displacement, $v(t) = \frac{dx(t)}{dt}$ and $a(t) = \frac{d^2 x(t)}{dt^2}$ are then obtained from differentiation and smoothing, using a Savistky-Golay filter. The parameters of the filter are manually adjusted by comparing the acceleration obtained from the motion capture to the signal from the accelerometers: for moderate tempo exercices, the order was set to 3 and the window size to 80ms.

As defined previously, each bow stroke is segmented based on the velocity zero crossings: a stroke starts and ends with null velocity. For each stroke, we compute:

- the used bow length $D = |x(t_{k+1}) - x(t_k)|$, where t_k are the instants of zero velocity,

- the movement duration $T = t_{k+1} - t_k$,

- the initial bow acceleration $a_0 = a(t_k)$,

- the maximum and minimum bow acceleration $a_{max} = \max\limits_{t \in [t_k; t_{k+1}]} a(t)$, $a_{min} = \min\limits_{t \in [t_k; t_{k+1}]} a(t)$.

These computed data are then used as empirical conditions to synthesize bow velocity profiles.

4.2 Synthesis of bow velocity profiles

For each stroke, three velocity patterns satisfying system (1) are synthesized using the computed bowing parameters. These models are:

- a trapezoidal model Tr, derived from minimum impulse solutions,

- a discrete minimum jerk model J_d,

- a cyclical minimum jerk model J_c.

The Tr pattern is a deformable trapezoidal shape. The initial and terminal slopes are determined by a_{min} and a_{max} and the constant peak velocity is computed taking into account bow length D, time T and the slopes. The full analytical expression of the model is derived from minimum impulse solutions and can be found in appendix C. Like minimum impulse solutions, this model minimizes the total impulse (time integral of the control action) over the stroke. However, the minimization is here computed based on conditions determined by the measured initial and terminal accelerations, instead of a constant maximum acceleration value. For this trapezoidal model, the velocity variations are described by the triplet a_{max}, 0, a_{min}.

The discrete and cyclical minimum jerk solutions, J_d and J_c respectively, minimize acceleration transients over the stroke (see section 2.3). These two solutions can be expressed as polynomials[1] and full analytical expressions are given in appendix C. The discrete model J_d and cyclical model J_c are computed taking into account the initial acceleration a_0, the maximum acceleration over the stroke a_{max}, the distance spanned D and the duration

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[1]The analytical expressions for minimum jerk models are generally found from dynamical optimization.

of movement T. In this process, an additional condition must be defined to comply with the unimodal constraint we chose: acceleration curves can not change sign more than once per stroke. We consider that the minimization has no solution when only non-unimodal solutions are possible with the measured data.

It must be noticed that when a_0 approaches the maximum acceleration over the stroke a_{max}, the velocity model of discrete minimum jerk J_d approaches the cyclical minimum jerk model J_c. This model can therefore account for continuity between discrete and cyclical movements (Nelson, 1983; Hogan and Sternad, 2007).

4.3 Comparison function

At this point, a comparison function determining which synthesized model is the most similar to the measured data is needed. In (Nelson, 1983; Ostry et al., 1987; Perkell et al., 2002), the velocity patterns are compared on the basis of their maximum peak velocity normalised by the average velocity. However, this single value may not be sufficient to determine convincingly similarity between velocity patterns. For example, on Figure 1, all velocity patterns have identical average velocity. The measured data (O) looks trapezoidal and therefore, based on its shape, the most similar model should be minimum impulse (I). However, looking at peak velocities, it appears that (O) should be associated with minimum energy (E), and not minimum impulse (I).

In this paper, we propose a different approach. We base our analysis on the entire profile by considering correlations between data and model curves, over each strokes. Correlation here accounts for linear relationships between the two curves and therefore stresses velocity pattern shape similarities. In this case, the measured data (O) is eventually associated with the minimum impulse model (I) as they are the most correlated. Moreover, since it was found in (Rasamimanana et al., 2006) that acceleration curves show characteristic properties depending on bowing techniques, we choose to carry out our analysis on acceleration profiles.

Figures 3 and 4 show the synthesized velocity patterns computed from the trapezoidal model and the minimum jerk model, their corresponding accelerations curves, and the measured data for strokes in *Détaché* and in *Martelé*.

On the graphs, one can observe that even when good match can be found between the data and the model, e.g. on Figure 4 between the minimum jerk model and data, there remains some differences. This can be explained by
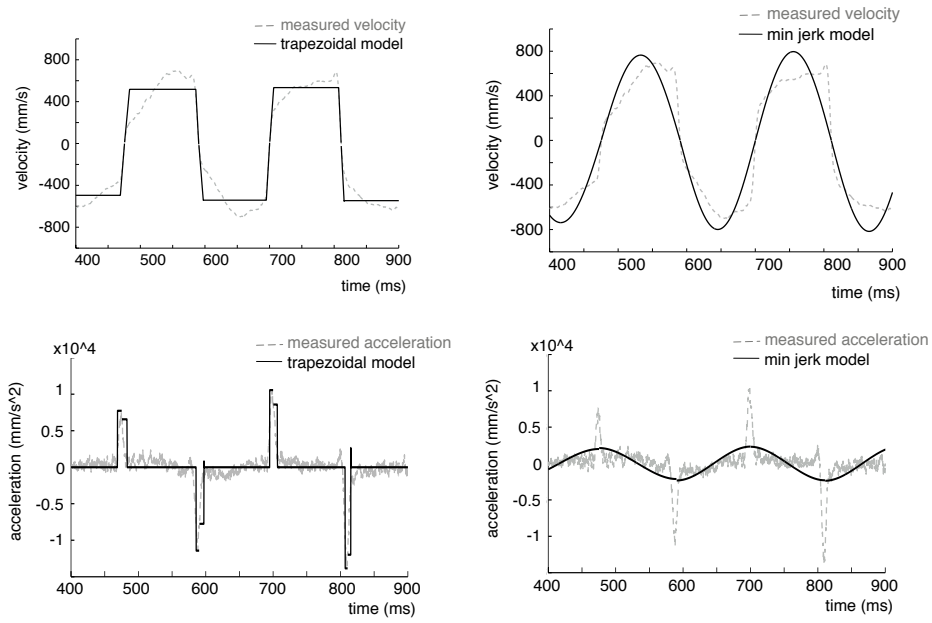
Figure 3: Trapezoidal and cyclical minimum jerk models for four measured *Détaché* strokes. Top plots show the measured and the synthesized velocity patterns. Bottom plots show the acceleration patterns. Positive and negative velocities show alternation between downbows and upbows
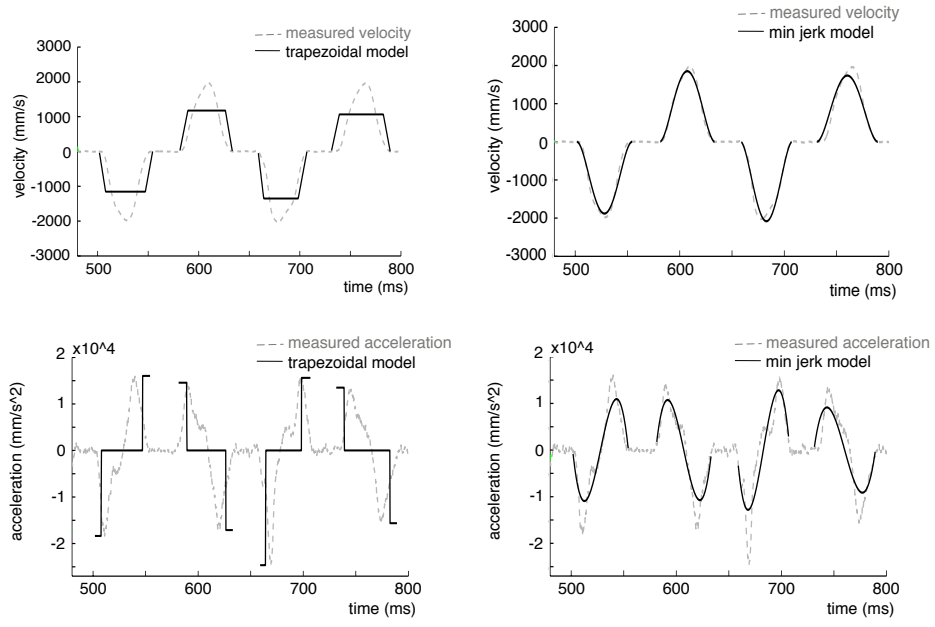
Figure 4: Trapezoidal and cyclical minimum jerk models for four measured *Martelé* strokes. Top plots show the measured and the synthesized velocity patterns. Bottom plots show the acceleration patterns. Positive and negative velocities show alternation between downbows and upbows

the fact that the cost based minimization represents only one of the different constraints that players must deal with when bowing. Another reason that could be pointed out is the assumption of unimodal velocity profiles, which imposes possible velocity variations to a single increasing phase and a single decreasing phase.

The comparison process is performed as follows. For each bow stroke, we correlate the acceleration curve to each of its synthesized versions (Tr, Jd and Jd). This hence gives a triplet of correlation factors that accounts for the similarity between a single measured stroke and its associated models. The model giving the highest correlation value corresponds to the most similar model. Besides, it should be noticed that by providing separate measures for all three models, this triplet actually reflects possible trade-offs between performance objectives, as introduced in (Nelson, 1983).

5 Results

This section begins with the analysis of velocity profile shapes in simple cases, i.e. isolated notes and scales, for the bowing techniques *Détaché* and *Martelé*. The results are then compared to more complex cases, such as mix of fast and slow rhythms, *accelerando / decelerando* and an exercise with mixed bowing techniques. The evolution of velocity profile shapes according to the bow stroke context is evaluated using the previously described method.

5.1 Single strokes and scales

In a first step, bow strokes are studied in simple, stereotypical cases involving the two bowing techniques *Détaché* and *Martelé*.

5.1.1 Single strokes

The first case corresponds to the performance of individual strokes separated with silence at a moderate tempo, namely quarter notes at 80 bpm, at a *forte* dynamic, interleaved with pauses ($Task1$). For each stroke, the three previously described velocity models are computed and compared to the measured data using the acceleration curves correlation. The results show that for all players, the most similar model to the measured separate strokes is the discrete minimum jerk model J_d, for both bowing technique *Détaché* and *Martelé*. These movements therefore appear to satisfy an optimal smoothness over the execution of strokes for both bowing techniques. This

result actually coincides with classical results on voluntary arm movements in one dimension, generally minimizing jerk over the task, such as described for example in (Hogan, 1984) in the case of reaching movements.

The comparison results are illustrated for one player on Figure 5 for *Détaché* strokes (top left) and for *Martelé* strokes (top right). On top, the measured velocity curve is displayed. The graphs below shows the plot of the correlation factors for the trapezoidal model Tr, the discrete minimum jerk J_d and the cyclical minimum jerk J_c according to each stroke. For each stroke, the highest correlation factor is highlighted (the stroke is surrounded by a rectangle). For both bowing techniques, the correlation factors of the trapezoidal model are much lower (< 0.6) than the correlation factors for discrete and cyclical minimum jerk (> 0.85). In the case of *Détaché*, the discrete and cyclical jerk models are particularly similar, as expressed by the correlation factors, showing that the initial accelerations are close to the maximum accelerations.

5.1.2 Scales

The second stereotypical situation corresponds to scales performed in *Détaché* and in *Martelé* at a moderate tempo (80 bpm) at a *forte* dynamic ($Task2$). This case allows for the study of possible differences between the performance of discrete, separate strokes and a continuous series of bow strokes. The results actually show interesting differences from single strokes.

For *Détaché*, the best model that describes the measured velocity is the trapezoidal model Tr as displayed on Figure 5 bottom left. This is different from the single strokes case where J_d represented the best model. For all three bowed string players, the correlation factors are indeed the highest for this model (with very high values between 0.85 and 0.9) compared to the others. It is also interesting to note that in this case, the discrete minimum jerk model J_d can not yield to any possible unimodal solutions for the strokes. The cyclical minimum jerk model J_c provides a less appropriate fit with the measured data with correlation factors around 0.5. In this case, the movements involved in the performance of *Détaché* are not optimally smooth anymore, but appear to be better explained with minimum velocity variations over the stroke.

To the contrary, for *Martelé*, the results are similar to the discrete case, as the model that best correlates with the data still is the discrete minimum jerk pattern J_d. However, as can be seen for *player*1 on Figure 5 bottom right, players generally stopped the bow between strokes when performing a *Martelé* scales. This short pause is indeed used to put some pressure on the
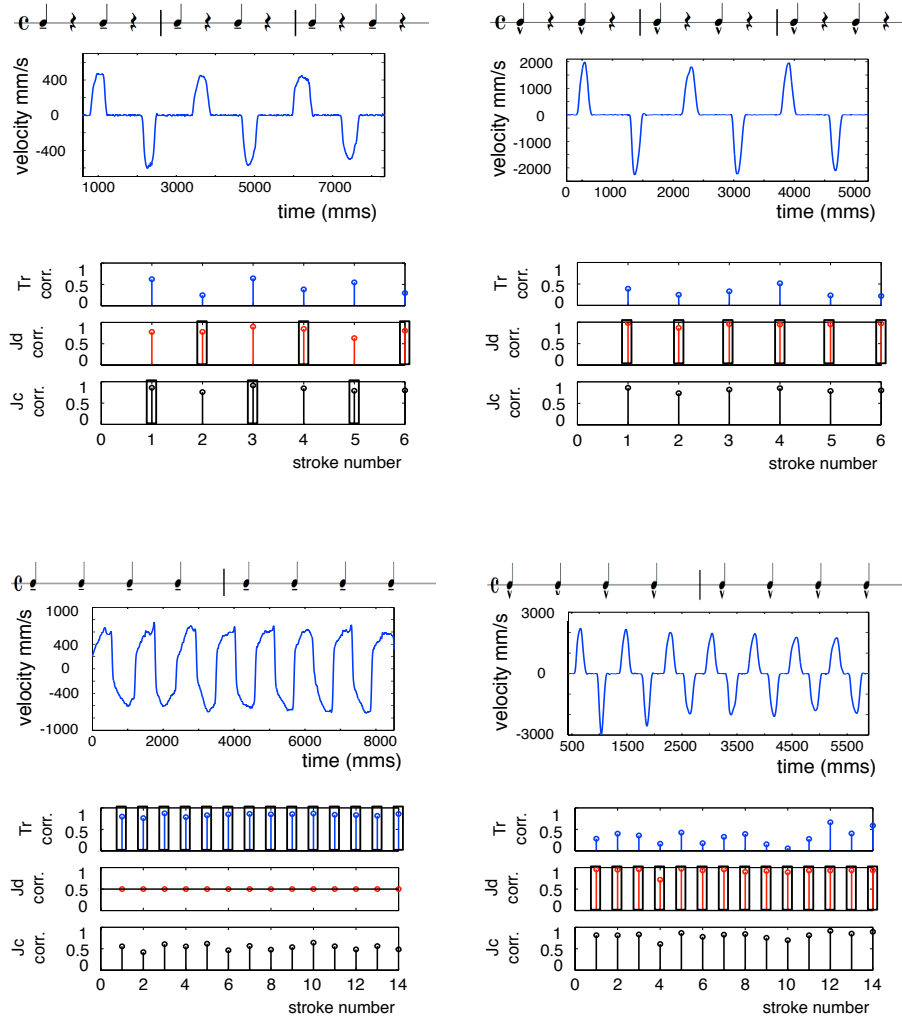
Figure 5: for all plots, from top to bottom: score, measured velocity patterns and correlation factors for the three models (Tr, Jc and Jd). For each stroke, the highest correlation factor is highlighted with a rectangle. Plots on the left correspond to *Détaché* and plots on the right correspond to *Martelé*.

bow before the beginning of strokes, a characteristic of this bowing technique (Demoucron et al., 2006): formally, these *Martelé* scales corresponds to a sequence of discrete movements.

The analyses of bow velocity profile shapes for these simple bow strokes are used as references to study the more complex situations, as described in the following sections.

5.2   Mixed bowing techniques

We consider in this section a mix of two types of bowing techniques, similarly to the study by Dahl (2000) on drums. The musical situation consists in a fragment of a violin study made of patterns of four quarter notes, with two *Détaché* and two *Martelé*, performed at a moderate tempo (80bpm) and at a *forte* dynamic ($Task3$). In the previous section, we found that, in the case of scales, *Détaché* is best described with the trapezoidal model, while *Martelé* is best described with the discrete minimum jerk model. Therefore, based on this result, the sequence of four quarter notes should lead to (Tr, Tr, J_d, J_d). The performed analysis however shows a different result as shown on Figure 6. For each stroke, statistics are reported over the 36 recorded patterns performed by 3 players. Figure 6 shows the percentage of each model for the four strokes of a pattern. The first, third and fourth strokes mainly corresponds to the expected models, i.e. Tr (76.5%), J_d (76.5%) and J_d (100%), respectively. Interestingly, the second stroke is best explained by the cyclical minimum jerk movement J_c (50%), instead of the expected trapezoidal movement Tr (26.5%). In other words, when considering a sequence of four *Détaché*, like in the case of scales, all bow strokes are best explained with movements minimizing the velocity variations (Tr). However, when mixing two *Détaché* and two *Martelé* in a sequence of four strokes, the second stroke appears to be affected by the models that was found to corresponds best to *Martelé*, i.e. optimally smooth movements. This can be viewed as a property of anticipatory behaviours.

Inversely, when considering a sequence of four *Martelé*, e.g. in the case of scales, the bow strokes are all best explained with discrete minimum jerk movements J_d, the bow generally being stopped before a *Martelé*. In this sequence of four strokes, however, it can be noticed that the third stroke, i.e. the first *Martelé*, is in some cases best explained by J_c (23.5%). This therefore indicates that in those cases, the bow is not completely stopped between the end of the *Détaché* stroke and the beginning of the *Martelé* stroke. This result could actually be interpreted as a phenomenon of persistence of the last *Détaché* stroke on the first *Martelé* stroke.
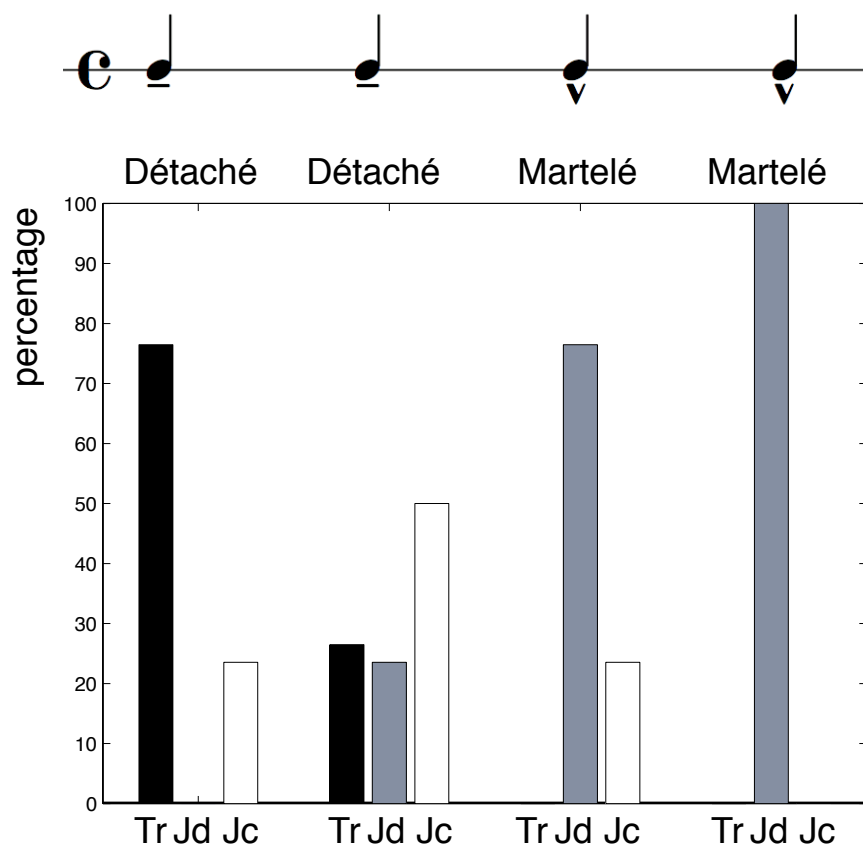
Figure 6: Mixed bowing techniques: percentage of each model for the four strokes of a pattern, consisting of two *Détaché* and two *Martelé*. Results include cases reversing *Martelé* and *Détaché*.

In speech, anticipation and persistence between phones are the two aspects that define *coarticulation*. It can be particularly interesting here to draw such a parallel between speech production and bowing movements. More specifically, coarticulation in speech is known to be essential for intelligibility, although not always directly audible. We can wonder whether this *gesture coarticulation* could contribute to a *musical intelligibility*, giving to a performance some organic properties that makes it humanly plausible, as opposed to purely computational processes.

5.3   Mixed rhythms and influence of stroke frequency

The following musical exercises extend the previous cases by the addition of different rhythms.

5.3.1   Mixed rhythms

The performers played a series a rhythmic patterns, consisting in a quarter note followed by four sixteenth notes, at a moderate tempo ($80 bpm$), at a *forte* dynamic level ($Task4$). This situation therefore creates a context mixing slow and fast rhythms. The same analysis is performed. Results are close for player1 and player2 as discussed below. Player3 is discussed separately.

Results for player1 and player2 indicate that the effort-based models actually depend on the rhythms. All quarter notes display a similar bow velocity profile that is best described with the trapezoidal model Tr, while sixteenth notes all have velocity profiles best described with the cyclical minimum jerk model J_c. Namely, the performance of this rhythmic pattern is best described with the sequence (Tr, Jc, Jc, Jc, Jc, Tr). This result actually suggests that different performance objectives underlie the execution of this sequence of rhythms. On quarter notes, players 1 and 2 tend to minimize the variation of velocity, producing a relatively constant sound, as opposed to sixteenth notes, where players 1 and 2 tend to produce the smoothest movement for the four of them. This result therefore invites to distinguish two *musical gestures* in their performance, i.e. one for the quarter notes and one for the sixteenth notes, which is actually found consistent from the players' viewpoint: intuitively, four sixteenths are not played as four times one sixteenth but rather as one unit. Figure 7 displays the correlation results for a performance of four rhythmic patterns. The results show patterns of one trapezoidal model and four minimum jerk models.

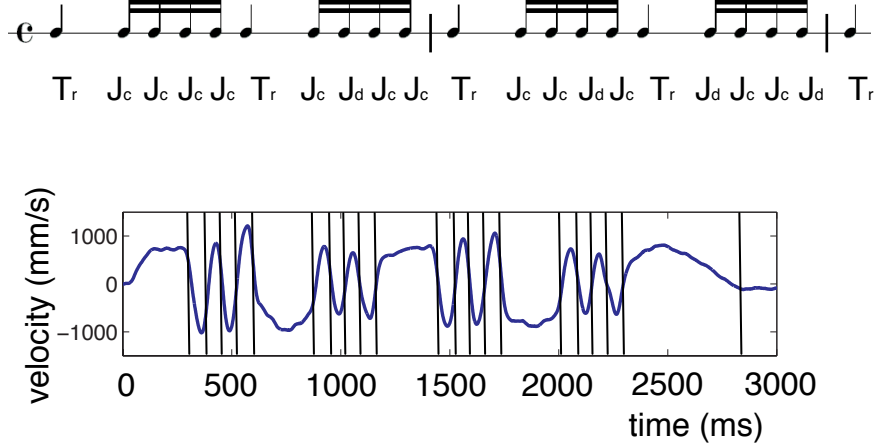Interestingly, the results for player3 are relatively different. The strokes

Figure 7: Mixed rhythm: score and measured velocity patterns. The best explaining model is given for each stroke.

are all closer to the cyclical minimum jerk model. In the light of our previous results, this could suggest that the player performed the sequences of rhythmic patterns as one whole. These results on the three players clearly show that in a musical context, the instrumentalists have different proper performance strategies. Such differences were not found in the case of single strokes or scales.

5.3.2 Accelerando / Decelerando

The next bow stroke context considers an *accelerando / decelerando*, i.e. a series of *Détaché* strokes performed with an increasing frequency followed by a series of strokes with a decreasing frequency ($Task5$). This case was already studied in the article (Rasamimanana et al., 2007), where different velocity patterns were reported and characterized with a sinusoidal non-linear fit. The analysis done in this article enables to further understand these results as displayed on Figure 8. As in the case of the above mixed rhythm sequence, slowest strokes are best described by the trapezoidal model and fastest strokes by minimum jerk models, therefore approving that the fastest movements tend to optimize acceleration smoothness and that the slowest movements tend to optimize velocity variation. Interestingly, the

frequencies of the changes from one cost to the other are similar to those reported in (Rasamimanana et al., 2007), i.e. around 8Hz.

Globally, this modelling confirms the existence of the transitions reported in (Rasamimanana et al., 2007). Moreover, the modelling presented here brings a new interpretation: the slow and fast parts of the *accelerando / decelerando* can be explained by different optimizations, from minimization of velocity variations to minimization of jerk.
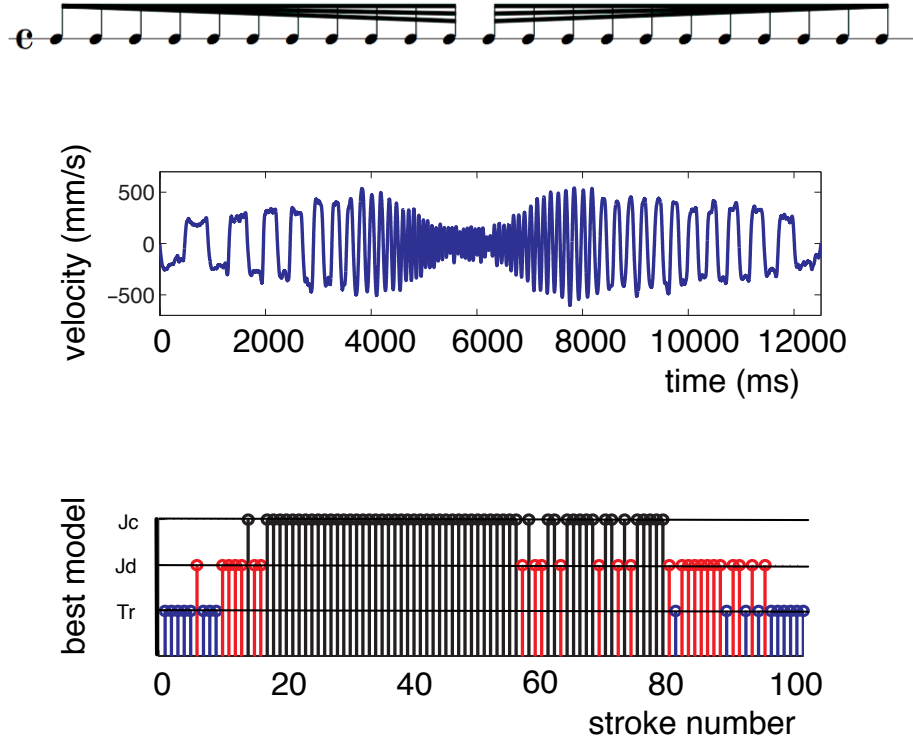

Figure 8: Accelerando / Decelerando: score, measured velocity patterns and best models (highest correlation).

6 Summary and perspectives

In this article, we considered effort costs associated with bowing movements to analyse possible strategies expert players use. With the help of a kine-

matic formalism, we related bowing techniques to different effort costs and based on this methodology, we evidenced the possible influence of the bow stroke context, in particular the influence between strokes.

The results on single strokes and scales show that the different bowing techniques can be modelled as different performance optimizations. Moreover, different optimizations also depend on tempo as shown with the *accelerando / decelerando*. At least two minimization costs are therefore relevant, namely jerk minimization and minimization of velocity variations. Nevertheless, notable differences between players are found depending on the musical context: mixing rhythms or bowing techniques. Such differences are actually expected since musicians might use different bowing strategies to play a given musical passage (Winold et al., 1994). The methodology we proposed in this paper seems to be able to take into account such variations, providing a framework for the interpretation of the underlying strategies. For example, we were able to evidence from a gesture perspective different groupings of notes related to phrasing. Moreover, the modelling presented in this article enabled to show anticipatory behaviour and coarticulation on bowing movements: the bowing movement of strokes is influenced by its neighbors, as occurring in speech coarticulation between phonemes. We believe that the evidence of such behaviour is important since it reveals a promising path to improve playability of digital musical instruments that usually do not take into account such effects.

More generally, the consideration of effort constraints can bring interesting insights to music performance analysis and synthesis. This approach can for example be useful for the definition of sound controls, still largely based on a MIDI/ADSR framework. The results presented in this paper indeed indicate that designing control as successive independent units is limiting compared to music playing on acoustic instruments. Moreover, this approach can also be useful to simplify the control of sound synthesis. Classes of temporal profiles related to physical costs can be intuitively controlled with only few input parameters.

7 Acknowledgements

A   Measures of physical cost

Five different performance costs.

$$\text{time cost:} \quad T = \quad \text{movement time,} \tag{2}$$

$$\text{force cost:} \quad A = \max_{t \in (0,T)} |u(t)|, \tag{3}$$

$$\text{impulse cost:} \quad I = \frac{1}{2} \int_0^T |u(t)| dt, \tag{4}$$

$$\text{energy cost:} \quad E = \frac{1}{2U} \int_0^T u^2(t) dt, \tag{5}$$

$$\text{jerk cost:} \quad J = \int_0^T \dot{a}^2(t) dt, \tag{6}$$

where, $u(t)$ is the control action, U is the control action limit, $a(t)$ is the mass acceleration, and T the duration of movement.

B Analytical solutions

In the case of negligible frictions, the analytical solutions to system 1 are:

minimum time:
$$V_t(t) = \begin{cases} Ut & \text{if t} \in [0; \frac{T_m}{2}[\\ -Ut & \text{if t} \in [\frac{T_m}{2}; T_m] \end{cases} \quad \text{where,}$$
$$T_m = \sqrt{4D/U},$$

minimum force:
$$V_f(t) = \begin{cases} A_m t & \text{if t} \in [0; \frac{T}{2}[\\ -A_m t & \text{if t} \in [\frac{T}{2}; T] \end{cases} \quad \text{where,}$$
$$A_m = 4D/T^2,$$

minimum impulse:
$$V_m(t) = \begin{cases} Ut & \text{if t} \in [0; \frac{V_0}{U}[\\ V_0 & \text{if t} \in [\frac{V_0}{U}; T - \frac{V_0}{U}[\\ V_0 - Ut & \text{if t} \in [T - \frac{V_0}{U}; T] \end{cases} \quad \text{where,}$$
$$V_0 = \frac{TU}{2} - \sqrt{(\frac{TU}{2})^2 - DU},$$

minimum energy:
$$V_e(t) = 6D/T^2 \, (t - t^2/T),$$

discrete minimum jerk:
$$V_{jd}(t) = A_0 t \left[1 + 6\left(\frac{5D}{A_0 T} - 1\right)\frac{t}{T} - 5\left(\frac{6D}{A_0 T^2} - 1\right)\left(2\frac{t^2}{T^2} - \frac{t^3}{T^3}\right)\right],$$
where A_0 is the initial acceleration.

C Velocity models

trapezoidal model:
$$V_{trap}(t) = \begin{cases} a_{max}\, t & \text{if t} \in [0; \frac{V_p}{a_{max}}[\\ V_p & \text{if t} \in [\frac{V_p}{a_{max}}; T - \frac{V_p}{|a_{min}|}[\\ V_p - |a_{min}|t & \text{if t} \in [T - \frac{V_p}{|a_{min}|}; T] \end{cases}$$
$$\text{where, } V_p = \left(\frac{1}{a_{max}} + \frac{1}{a_{min}}\right)\left(T - \sqrt{T^2 - 2D(\frac{1}{a_{max}} + \frac{1}{a_{min}})}\right),$$

discrete minimum jerk:
$$V_{jd}(t) = A_0 t \left[1 + 6\left(\frac{5D}{A_0 T} - 1\right)\frac{t}{T} - 5\left(\frac{6D}{A_0 T^2} - 1\right)\left(2\frac{t^2}{T^2} - \frac{t^3}{T^3}\right)\right],$$
where A_0 is the initial acceleration.

cyclical minimum jerk:
$$V_{jc}(t) = 5D/T^2 \, (t - 2\frac{t^3}{T^2} - \frac{t^4}{T^3}),$$

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