Psychophysical Validation of a Proprioceptive Device by Cross-Modal Matching of Loudness

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Summary
A large number of studies performed by S. S. Stevens, J. C. Stevens and their collaborators have estimated the intensity of one sensory modality by way of another one related to the proprioceptive sensation of muscular force and limb position. As such, using a cross-modal matching procedure, it should be possible to associate a proprioceptive sensation with an auditory sensation (e.g. loudness, brightness, roughness) having an equivalent strength. A new proprioceptive estimation device for the continuous unidimensional judgment of non-stationary sounds has been developed, and in this paper we establish an individual calibration method and the loudness scale of stationary, 1-KHz pure tones by a cross-modal matching paradigm using the new device. The loudness scale and the proprioceptive scale related to muscular force applied to a unidimensional joystick with force feedback were evaluated independently by direct estimation techniques: ratio and magnitude production, absolute magnitude estimation. The proprioceptive function obtained is characterized by a power function with an exponent of 1.77. The results obtained with ratio scaling and cross-modal techniques are similar. However, the set of results obtained with the cross-modal technique appear more stable and less dependent on experimental conditions. Finally, these results allow us to envisage the use of this device for different auditory assessment applications.

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1. Introduction
For sounds lasting a few seconds or considered stationary, traditional procedures of subjective evaluation [1, 2] and measurement [3] have been used. They yield reliable and robust results. For example, several experiments [4, 5, 6, 7, 8, 9] have been performed to evaluate the loudness of 1-KHz pure tones based on techniques introduced by S. S. Stevens in the 1950s. The most well-known are the magnitude estimation and production techniques. These with, S. S. Stevens produced the same scale corresponding to a measure of perceived acoustic intensity. Several experiments performed with these direct evaluation techniques have given the power law relating physical to perceptual magnitudes proposed by S. S. Stevens [10]:

\[ \Psi = k\Phi^\alpha, \] (1)

where \( \Psi \) corresponds to subjective intensity, \( \Phi \) corresponds to physical intensity, \( k \) is an arbitrary constant to adjust the scale, and \( \alpha \) is the exponent that depends on the sensory modality and conditions of stimulation. For example, the psychophysical function obtained that expresses the sensation of acoustic intensity as a function of physical intensity of a 3-KHz pure tone, corresponds to a power function with an exponent of 0.67. Experiments performed by S. S. Stevens in different sensory domains have given exponents varying from 0.33 (visual brightness) to 3.5 (electric shock). The power law seems to be applicable without exception to any physical continuum involving a variation in sensory intensity, at least for values not too close to threshold where certain adjustments are often necessary (cf. [11] for loudness). The construction of direct scales can thus be reduced to determining the exponent of the power function. However, the problem with direct scaling resides in the ability of the subject to detect and transmit the right ratios corresponding to his or her sensations. A review of the psychophysical literature concerning ratio-scaling techniques [11, 12, 13, 14, 15] reveals a variability of the results in the subjects’ judgments according to the type of technique, the level or the value of the reference sound, the presence or absence of a reference sound, and the range of stimuli presented, among other factors. S. S. Stevens [16] proposed a cross-modal matching (CMM) method for confirming the power law relation which does not require the subject to make numerical judgments. The task consists in matching two sensations coming from different sensory modalities, one of which has been calibrated beforehand by a direct estimation method [16, 17, 18].

The use of this CMM method with an adequate force-feedback proprioceptive device has three main advantages. Firstly, as S. S. Stevens emphasized, on the basis of knowledge concerning the function for a sensory modality, it is...
possible to determine the psychophysical functions of other sensory modalities by cross-modal matching. For example, the psychophysical functions of different auditory attributes (loudness, roughness, sharpness) can be obtained by way of the proprioceptive function corresponding to the assessment device. Secondly, the data in the literature reveal variability, between and within methods [11, 12, 13, 14, 15] and among subjects, that can be reduced by determining for each subject an individual matching function. It is necessary with this aim in mind that the assessment device be individually calibratable as a function of different factors such as individual sensitivity that varies with the stimulus range and the individual process of associating the two sensory modalities. Thirdly, the intuitive and implicit nature of this assessment procedure in comparison with other methods of numerical estimation or ratio estimation suggests its possible uselessness in procedures where rapid assessment is necessary. For example, in the case of continuous judgment of nonstationary signals, it is important to use a method that transcribes rapidly, easily, and continuously the subjects' estimates. Further, it is interesting that this method can provide subjects with continuous information concerning their ratings by providing force feedback.

The aim of the present article is to test the psychophysical reliability of CMM with a new proprioceptive device developed for that purpose. We limit this study to determining the loudness function of 1-kHz stationary pure tones, in order to compare our results with those of the literature and to validate our device with the cross-modal matching technique. In a future article [19], this technique will be applied to time-varying sounds and compared with other techniques. The proprioceptive evaluation device developed for this work is presented in section 2. In the first series of experiments (section 3), we determine the loudness scale with ratio and magnitude production, as well as with magnitude estimation. To obtain greater precision, we have used different methods to examine the biases due to each one. Similarly, we have repeated these tests with different levels and values for the reference sound in order to observe their influence on the evaluation. We then determine the psychophysical function related to proprioceptive sensation with the ratio production method, on the one hand, and with magnitude estimation, on the other hand. These functions were determined for both directions of manipulation of the device: forwards with increasing force and backwards with decreasing force (section 4). The aim is to find a relation between proprioceptive sensation and the physical magnitude of the force exerted on the hand with a direct method using the proprioceptive device. In other words, we sought to determine the ratio scale of apparent force exerted by a subject on this device. Finally, we compare the psychophysical functions for proprioceptive and auditory modalities obtained with independent methods to the function obtained directly by cross-modal matching (section 5). The cross-modal function was also determined for both directions of manipulation of the device.

2. Technical description of the proprioceptive device

2.1. Weight-related resistance: device schema

In order to evaluate auditory sensation by a sensation of muscular resistance, a proprioceptive system was developed. This system acts on the sensitivity of muscles, bones, ligaments, and joints, and provides information concerning the equilibrium and position of limbs of the body in space. The transmitted information and sensitivity depend on the type of effort and the movement provoked by the system. It is thus important in the choice of the device to specify what type of effort is involved in the subject’s task during the experiment. In other words, the subject must know to which gestural category corresponds the required muscular effort as a function of the device. The representation of the effort to be made is different according to the manipulated object, and to each object is associated an appropriate scale of muscular effort. For example, the scale will be different if one squeezes on a water balloon or if one manipulates the handle of a pick-axe. The schematic of the device we settled on is shown in Figure 1.

With this system, the subject has information on the evaluation being made by force feedback and the position of arm and shoulder. The lever exerts a resistance as a function of the angle of displacement, the effect of which is to create a proprioceptive sensation. As such, the subject associates a combination of muscular force and displacement with any intensity variation perceived by moving the lever of a "joystick" in the direction corresponding to the auditory sensation. For example, to an auditory sensation for which the intensity is perceived as being two times greater, the subject doubles the proprioceptive sensation. In this way, the result related to an intensity sensation does not depend on the ability of the listener to estimate the numerical magnitude related to the sensation, which involves a sensory-to-conceptual matching process, but rather to match one sensory magnitude to another. The data obtained allow us to establish a relation between the muscular force in Newtons and the sound pressure level in dB SPL.

The displacement of the lever is recorded with the help of a 10 kΩ potentiometer mounted in the axis of the lever motion. It is configured such that the output voltage is a linear function of the lever angle. The electric voltage is transformed into MIDI data and read by the computer program running the experiment.

2.2. Equation of the system

The variables in the system include the following: mass \( m \), distance \( l \) of the mass from the axis of rotation \( oz \), the distance \( e \) of the handle from the axis of rotation, and the angle \( \alpha \) between the shaft and the vertical axis \( z \). For the experimental procedures presented in this article, the mass-shaft-subject system effects a rotational movement that is quasi-stationary at each instant \( t \). The sum of the moments of force applied to the system is thus zero. The modulus of the force as a
function of the angle is given by

\[ R = \frac{mg}{\sin \theta} \]

Note that \( R \) is a force tangent to the movement applied by the subject in the direction of displacement of the lever and \( P \) is the force applied by the mass \( (P = mg) \).

### 3.1. Subjets

A group of 10 subjects (8 men and 2 women) with ages between 25 and 30 years participated in all of the tests. Ten additional subjects (5 men and 5 women, aged from 25 to 39 years) only participated in the magnitude estimation of the proprioceptive sensation and the cross-modal matching procedure. Two subjects were experienced in psychoacoustic testing and the others had never participated in this kind of test. Each subject had a training session to become familiar with the experimental task. No subject reported having hearing problems.

### 3.2. Apparatus and stimulus generation

In each experiment, the stimuli used were 1-kHz, stationary sine tones sampled at 44.1 kHz with 16-bit resolution. They were generated digitally on the IRCAM Musical Workstation [20] comprising a NeXT computer, an ISPW digital signal processing card, and MAX software. The sounds were then converted by ProPort DACs with integrated anti-aliasing filters and amplified by a Canford stereo amplifier before being presented diotically over AKG-1000 headphones. Subjects were seated in a Soluna S1 double-walled sound isolation booth. Sound levels at the headphones were measured with a Brüel and Kjær 2209 sound-level meter. The experiment was run using the PsiExp experimental software environment [21]. For Experiments 1 and 2 (ratio and magnitude production, respectively), two 1-s tones with 40-ms linear attack and decay ramps were presented successively, initially with the same intensity, and were separated by a silent interval of 1 s. Repeated successive presentations of the tone pair were separated by a 2-s silence. The first tone was the reference and the second was to be adjusted in level with a cursor on the computer screen that was moved with a mouse. The reference and variable sounds were repeated until the subject indicated satisfaction with the adjusted level. In Experiment 3 (magnitude estimation), stimulus presentation was similar but the level of the second sound was different from that of the reference and could not be modified.

### 3.3. Experiment 1: Ratio production

#### 3.3.1. Procedure

Subjects were asked to modify the level of the second sound of the repeating pair so that it would be twice as loud as the reference sound. At the beginning of the experiment, the two tones were presented at equal intensity levels for 1 s. Subjects were asked to adjust the level of the second tone until it was judged to be twice as loud as the reference sound. This process was repeated until the subject indicated satisfaction with the adjusted level. The reference and variable sounds were repeated until the subject indicated satisfaction with the adjusted level. In Experiment 3 (magnitude estimation), stimulus presentation was similar but the level of the second sound was different from that of the reference and could not be modified.
Figure 3. Averageloudness function obtained by magnitude production for different reference levels: (a) 50 dB SPL, (b) 60 dB SPL, (c) 70 dB SPL. Asterisks indicate means across subjects and vertical bars represent ± 1 standard deviation. A modulus of 10 was associated with the reference level.
Figure 4. Loudness function obtained by magnitude estimation for different reference values: (a) 50 dB SPL, (b) 60 dB SPL, (c) 70 dB SPL. Asterisks indicate means across subjects and vertical bars represent ±1 standard deviation. A modulus of 10 was associated with the reference level.

3.5. Experiment 3: Magnitude estimation

3.5.1. Procedure

Once again sounds were presented in pairs. The first was the reference sound to which a numerical value of 10 was attributed. The second was presented at a different level. The levels included 30, 35, 45, 55, 65, 70, 80, 85, and 90 dB SPL. The order of presentation was randomly chosen for each subject. The subject entered on the keyboard a numerical value corresponding to the loudness of the second sound, such that the ratio of the numbers corresponded to the ratio of the loudnesses, i.e., if the second sound seemed twice as loud, a value of 20 should be entered. Once the value was entered, the next pair was presented, the reference level always being the same. For each estimate, the level and corresponding numerical value of the comparison sound were recorded.

In order to evaluate the influence of the level of the reference sound on subjects' responses, different reference levels were used: 50, 60, and 70 dB SPL. For each reference level, the experimental procedure was repeated.

3.5.2. Results

The results obtained with this method show once again that loudness is a power function of acoustic pressure. The psychophysical functions thus obtained are the means of the magnitude estimations over the ten subjects as a function of acoustic pressure expressed in dB (Figure 4). The functions vary again with the reference level, giving slopes of 0.35, 0.40, and 0.40 for 50, 60, and 70 dB SPL, respectively. The mean slope is 0.38 with a standard deviation of 0.03.

3.6. Discussion

The loudness functions obtained with the different methods all respect a power-law relation. This result holds for both global and individual data. However, a great deal of variability exists between individuals and between methods. Production of ratios of two and magnitude production give similar results, the exponent being 0.55 for both. Comparatively, the magnitude estimation method gives a shallower loudness function with an exponent of 0.38. A review of the literature reveals that results are variable according to the different options possible in the experimental procedure. S. S. Stevens has demonstrated that various factors can modify the form of the loudness function [6]. For example, the presence of a reference sound reduces the slope. This effect varies with the level of the reference and the number of stimuli presented around the reference value. Other research conducted by Hellman and Zwislocki confirms this result [7, 8, 9, 11]. In addition, results obtained by magnitude estimation generally underestimate the slope of the loudness function, as we found in the present study.
Nevertheless, a stable loudness function has been obtained from different methods: Robinson [5] and S. S. Stevens [6] with estimation, Scharf and J. C. Stevens [22] with ratio production and estimation, Feldtkeller, Zwicker and Port [23] for ratio production, Hellman and Zwischen [7] with estimation and magnitude production. The mean of these data gives a power function with exponent 0.54 for levels above 30 dB SPL. This result is in agreement with what we have found for ratio and magnitude production. As such we will retain this slope value for comparison with the rest of this study.

4. Force scaling

4.1. Experiment 4: Ratio production

4.1.1. Procedure

At the beginning of a trial, the lever of the proprioceptive device was positioned at 90° with respect to the horizontal plane (Figure 5). The subject initiated the experiment by clicking on a button on the computer screen and then displacing the lever to position “A” corresponding to a reference proprioceptive sensation. The reference position was indicated to the subject by a weak sound signal (40 dB, 400 Hz) that was present when the lever was within an angle of 1.6° centered on the reference. The subject’s task then consisted of displacing the lever from this position until the force sensation was doubled compared to the reference position. It was suggested to subjects to return to position “A” and then back to position “B” to verify the relation between the sensations, but to not do this more than two times. Once the correct position “B” was found the lever was to be held in that position until the key “V” was pressed on the keyboard to record the angle. Then the lever was returned to the resting position and an obligatory rest period of 5 seconds was imposed to avoid the effects of muscle fatigue. Position “B” then became position “A” and the procedure was repeated iteratively until the subject attained the upper limit of the system, a 90° angle, which was indicated by pressing the “L” key on the keyboard.

In order to evaluate the influence of the reference position on subjects’ responses, different angles were used: 16°, 40°, 5°, 28°, and again 16°. These reference positions corresponded to a pushing effort. To compare the sensation of pushing to that of retaining the lever and reducing the angle, two other reference positions were used: 86° and 67°. Starting from these latter reference positions, the subject was required to find a position for which the muscular effort was half that of the reference position, and the procedure was repeated iteratively until the lower limit of the system was reached at 0°. In all cases, a value of apparent force of 1 was assigned to the initial position, followed by values of 2, 4, 8, etc. for pushing mode and 0.5, 0.25, 0.125, etc. for retaining mode.

4.1.2. Results

A curve was obtained corresponding to the ratios of sensations as a function of the physical values recorded (angular displacement or the corresponding force that is entirely determined by the angle for a given mass and distance of the mass from the origin). Presented on log-log coordinates, seven linear functions corresponding to the different initial positions were obtained, indicating that the data nicely fit a power function. Individual differences between subjects, as shown in Figure 6, suggest that different strategies were used. Globally, the slopes are greater than 1, signifying that the proprioceptive sensation is positively accelerated as a function of the magnitude of physical force. This value increases for some subjects with an increase in the initial force. Further, the mean slopes for the proprioceptive functions in pushing and retaining modes are slightly different. The psychophysical function of proprioceptive sensation as a function of force (in Newtons) derived from the mean slope across all subjects corresponds to a power function with an exponent of 2.13 (s.d. = 0.7) for pushing and 1.50 (s.d. = 0.5) for retaining. In both cases, the function is obtained by doubling and halving, respectively, the evoked sensation with a different starting angle. Each of the two methods has an experimental bias of similar amplitude and opposite sign. In order to eliminate this bias, the mean of the two values is taken. The proprioceptive function obtained corresponds to a power function with an exponent of 1.81.

4.2. Experiment 5: Absolute magnitude estimation

4.2.1. Procedure

Subjects were asked to rate the sensation evoked by a given force on a numerical scale with a value proportional to the strength of the proprioceptive sensation. No reference value was given. Eight positions of the lever, each corresponding to a different force, were administered to subjects successively by a weak sound signal. In a first part, ten subjects started from the resting...
Figure 6. Proprioceptive psychophysical functions obtained for production of ratios of two and one-half for different initial positions. All initial positions are assigned an apparent force value of 1. Data are shown for four representative subjects: (a) subject 4 (pushing force, mean $a = 1.59$; retaining force, mean $a = 1.68$), (b) subject 5 (pushing force, mean $a = 3.23$; retaining force, mean $a = 1.11$), (c) subject 8 (pushing force, mean $a = 2.25$; retaining force, mean $a = 1.19$), (d) subject 10 (pushing force, mean $a = 3.34$; retaining force, mean $a = 2.69$).

For each subject, the predetermined positions were presented in a random order. The ten other subjects performed the two parts in reverse order.

4.2.2. Results

Figure 7 shows the mean ratings obtained for the set of 20 subjects. Here again, the slope for the retaining function (1.67) is lower than that for the pushing function (1.80). However, the difference between the two is smaller than that obtained by the ratio production method. On the other hand, one should note that there is a slight departure from linearity in log-log coordinates for the pushing function. The proprioceptive function determined by magnitude estimation corresponds to a power function with a mean exponent of 1.74. Note that ten of these subjects had performed the ratio production task (Experiment 4, section 4.1) 15 months earlier. Given the time between the two experiments, they cannot be considered to be influenced in the present task by their experience in the earlier one.

4.3. Discussion

The predicted result, on the basis of work by S. S. Stevens, J. C. Stevens and their collaborators [24,25,26], is a power function with an exponent greater than 1. S. S. Stevens measured the apparent muscular force exerted by a subject on the handle of a dynamometer using different methods of direct judgment. The obtained function respected a power law with an exponent of 1.7. In another experiment, a force was applied to the palm of the subject’s hand, yielding an exponent of 1.1. Similarly, a task consisting of displacing weights of different magnitudes produced a sensation that evolved according to a power function with an exponent of 1.45. The mean values of 1.74 and 1.81 determined with the new device by magnitude estimation and ratio production, respectively, are similar to that obtained by S. S. Stevens (1.7) with the handgrip system. Finally, the mean value of the exponent of the power function for proprioceptive sensation corresponding to our device is 1.77.
5. Experiment 6: Cross-modal matching

5.1. Individual calibration of the device

There are important differences between individuals that require a personalized calibration of the device: physiological differences related to muscular strength and sensitivity and differences in response strategy and auditory representation of the stimulus. A standard calibration procedure consists in the strategy clearly consisted in matching angular displacement; that is, he associated a given sound with two identical positions of the lever having different resistive forces. His judgments with respect to the maximal range of displacement for each mass-distance pair are superimposed. The three curves representing the variation in force as a function of the force intensity, obtained in pushing (circles) and retaining (squares) modes. Vertical bars represent ±1 standard deviation.

Examination of the curve in Figure 8 leads to the elimination of the stimulus used for the highest levels when the resistance of the system was low. The dash-dotted line presents a flattening at maximal displacement (90°) starting at 80 dB SPL. There are important differences between individuals that require a personalized calibration of the device: physiological differences related to muscular strength and sensitivity and differences in response strategy and auditory representation of the stimulus.
rather than muscular force to the perceived loudness. The judgments obtained for another subject were saturated for the highest levels when the global resistance of the system, corresponding to one of the three mass-distance pairs, was the weakest. Finally, for each subject, we recorded the most appropriate mass-distance pair and used it in the main matching task below.

5.2. Stimuli

Eight 1-kHz pure tones were used with levels of 40, 45, 55, 60, 65, 70, 80 and 85 dB SPL. The duration of the sounds was 1 s.

5.3. Procedure

The subject started the experiment by pressing a button. A sound of constant level was presented once every two seconds. First, from the rest position (angle 0°), the subject displaced the lever of the device until the apparent force was equivalent to the loudness of the sound and then, while holding the lever at that position, pressed the key "V" on the keyboard to enter the response. The procedure was repeated for each sound level. Then, in a second step, the experiment was done again but with a starting point from the maximum.

Figure 9. Psychophysical function obtained by cross-modal matching. The subject started the experiment by pressing a button. A sound of constant level was presented once every two seconds. First, from the rest position (angle 0°), the subject displaced the lever of the device until the apparent force was equivalent to the loudness of the sound and then, while holding the lever at that position, pressed the key "V" on the keyboard to enter the response. The procedure was repeated for each sound level. Then, in a second step, the experiment was done again but with a starting point from the maximum.

The order was reversed for the ten subjects. For each adjustment, the sound level was determined. According to Stevens' power law

\[ \log F = a \log I + k \]

where \( F \) is force and \( I \) is stimulus intensity on the matched dimension. The exponent \( a \) corresponds to the power function between the physical data for each stimulus. Finally, the slopes of the line describing the relation on log-log coordinates were determined. According to Stevens' power law

\[ \frac{F}{P_{\text{equivalent}}} = k \]
determined independently by ratio scaling methods based on numerical estimation.

The results obtained in the present study confirm firstly that loudness and the proprioceptive sensation being measured can be described by power functions. They also confirm that the slope (0.29) of the matching function obtained directly by CMM using the new proprioceptive device is very close to the slope that would be predicted from the power functions obtained in the previous experiments for loudness and proprioception independently (0.55/1.77 = 0.31). Note that the linear regression coefficients for the individual cross-modal matching functions vary between 0.90 and 0.99 in pushing mode and between 0.87 and 0.99 in retaining mode.

6. General discussion

With the help of a questionnaire administered at the end of each experimental session, we attempted to discern the level of difficulty of each scaling method and the strategies adopted by the subjects. At first, all methods seemed to baffle subjects in general, especially the cross-modal matching paradigm. However, after a few trials, they rapidly adapted to this latter procedure and judged it to be easier to perform compared to ratio and magnitude production and magnitude estimation. They all felt in the end that they responded instinctively; that is, they no longer proceeded by a succession of back-and-forth movements around a mean value, but made their judgment with a single movement, a rapid and direct positioning of the lever at the desired position with its corresponding apparent force that was perceived to be equivalent to the perceived loudness.

Globally, loudness varies as a power function of acoustic pressure with an exponent of 0.55. The psychophysical function for the proprioceptive device corresponds to a power function with an exponent of 1.77. The ratio of exponents for loudness and proprioceptive psychophysical functions give a predicted slope of the matching function of 0.31. The value obtained directly by cross-modal matching was 0.29, which we consider to validate both the device and the matching method for evaluating loudness scaling. Indeed these results

It is important to note that the results obtained with the cross-modal matching technique are less dependent on experimental conditions than those provided by direct estimation techniques. For example, the loudness functions determined by magnitude estimation and production are clearly dependent on the level of the reference sound. Note that a cross-modal matching experiment performed by Bond and S. S. Stevens [27] gives identical results for a group of five-year-old children and a group of adults. This suggests that the matching method does not depend on the ability to translate sensations into numerical values. The cognitive load induced by this method appears lower than that reported by our subjects for direct estimation methods. Judgments are likely to be the result of basic-level processing of sensory systems and are less dependent on individual strategies. This last point explains without a doubt why this technique allows us to obtain stable data in a short time after a limited period of familiarization, and further suggests that such a device would be reliable for evaluation of time-varying sound events, a subject to be addressed in a subsequent paper [19].

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References


1-kHz pure tone by a linear transformation, in log-log coordinates, of the associated proprioceptive scale.

This device would appear to be useful for scaling sensations corresponding to a variety of auditory attributes, for example, the sensation of loudness.

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