# Phase Unwrapping on the Sphere for Directivity Functions and HRTFs

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## Introduction

Applications using HRTFs or other directivity functions are often combined with interpolation techniques because measurements are only available at discrete directions with finite angular spacing. Linear interpolation, however, creates interference at high frequencies where the phase differences between the measurements are large due to time-delays. For HRTFs, interference is typically minimized by removing the all-pass components and separately interpolating the minimum-phase parts and the time-shifts (linear phase). To preserve the all-pass components, more general strategies linearly interpolate magnitude and unwrapped phase separately in the frequency domain. The unwrapped phase is related to the integrated group delay; thus interpolation is performed on group delay and magnitude separately, which seems suitable for HRTFs. However, spectral phase unwrapping is often degraded by error propagation along the frequency axis when applied to real measurements.

We present an alternative solution based on spatial phase unwrapping for each frequency, i.e. unwrapping of the phase across the spherical grid of directional measurement data.

## Absolute phase and principal argument

Any frequency response regarded at the frequency $\omega$ and direction $\boldsymbol{\theta}$ can be decomposed into magnitude and phase $H(\omega,\boldsymbol{\theta}) = |H(\omega,\boldsymbol{\theta})|\mathrm{e}^{\mathrm{i}\,\Phi(\omega,\boldsymbol{\theta})}$. The frequency response is invariant under addition of integer multiples of $2\pi$ to its phase and therefore one usually only regards the *principal argument* of the phase in the interval $]-\pi,\pi]$. It is related to the absolute phase by the principal argument function $\mathcal{W}$: $\phi(\omega,\boldsymbol{\theta}) = \mathcal{W}(\Phi(\omega,\boldsymbol{\theta})) = \mathrm{mod}_{2\pi}(\Phi(\omega,\boldsymbol{\theta})+\pi)-\pi$. The principal argument is wrapped into this interval and is therefore discontinuous over both the frequency and the measurement direction. By contrast, the absolute phase is related to the group delay by $\tau(\omega,\boldsymbol{\theta}) = -\frac{\partial}{\partial\omega}\Phi(\omega,\boldsymbol{\theta})$ and must be continuous over both $\omega$ and $\boldsymbol{\theta}$ in a physical sense. Reasonable interpolation therefore requires phase unwrapping, i.e. the retrieval of the absolute phase. In practice, this is done using the available discrete frequencies $\omega_k$ and discrete directions $\theta_l$.

## Phase unwrapping over frequency

Phase unwrapping/continuation over the frequency is defined as

$$\hat{\Phi}(\omega_k,\theta_l) = \mathcal{W}\{\Delta_k\phi(\omega_k,\theta_l)\} + \hat{\Phi}(\omega_{k-1},\theta_l), \qquad (1)$$

with $\Delta_k\phi(\omega_k,\theta_l) = \phi(\omega_k,\theta_l) - \phi(\omega_{k-1},\theta_l)$. Eq. (1) removes discontinuities, i.e. phase differences $|\Delta_k\phi(\omega_k,\theta_l)| > \pi$, between adjacent frequencies by adding integer multiples of $2\pi$.

This is the usual approach for absolute phase recovery in audio applications which enforces spectral phase continuity. Due to the recurrence relation in eq. (1), possible unwrapping errors accumulate along the frequency axis. This yields severely corrupted results if the low-frequency phase is noisy or inconsistent due to post-processing artifacts. Between neighboring directions, the unwrapping error causes discontinuities that appear to be random.

## Phase unwrapping on the sphere

Phase continuation over the measured directions can be defined as

$$\hat{\Phi}(\omega_k,\theta_{l'}) = \mathcal{W}\{\Delta_l\phi(\omega_k,\theta_{l'})\} + \hat{\Phi}(\omega_k,\theta_l), \qquad (2)$$

with $\Delta_l\phi(\omega_k,\theta_{l'}) = \phi(\omega_k,\theta_{l'}) - \phi(\omega_k,\theta_l)$. Unlike before, here the path of unwrapping needs to be defined. First, indices of all neighboring positions $\theta_{l'}$ of $\theta_l$ are revealed using *Delaunay triangulation* [2], which can be efficiently calculated using the Quickhull algorithm [1]. The relation given in eq. (2) removes discontinuity, i.e. phase differences $|\Delta_l\phi(\omega_k,\theta_{l'})| > \pi$, between neighboring positions in space by adding integer multiples of $2\pi$. This is only valid if the unknown absolute phase does not change by more than $\pi$ between neighboring positions.

A priori information is used to develop a spatial unwrapping path. The choice of the unwrapping path strongly influences the quality of the result as errors accumulate along this path. Therefore, it should run through reliable positions first and end at potentially corrupted ones. As a criterion for the reliability we used the SNR. Considering the spatial energy distribution of HRTFs, the SNR is approximately rotationally symmetric with respect to the interaural axis, decreasing from the ipsilateral to the contralateral side. Further, the group delay is known to increase in the same direction due to the increasing lengths of the acoustic paths, i.e. the absolute phase decreases. This additional a priori information is used to stabilize the results of the following algorithm.

**Algorithm.**

The spherical phase unwrapping algorithm for HRTFs is described as follows (see also [3]):

A) *Set first reference position.* Choose the position closest to the interaural axis on the ipsilateral side.

B) *Unwrap neighbors.* Unwrap phase change between the reference position $\theta_l$ and its neighbors $\theta_{l'}$ according to eq. (2).

C) *Smooth results.* If the unwrapped phase does not decrease with increasing lateralisation, subtract $2\pi$ until it

does.
D) *Set next reference position.* Constraints: Position has (i) already been unwrapped, (ii) at least one remaining wrapped neighbor, (iii) the least phase discontinuities to already unwrapped neighbors, and (iv) the smallest possible lateral difference to first reference position.
Repeat B, C and D until all positions have been unwrapped.
This algorithm iteratively unwraps the phase from the ipsilateral to the contralateral side due to the choice of the starting position in A) and the constraint D(iv) for the choice of the next reference position (see fig. 1). It also uses the a priori absolute phase information in C) to force the unwrapped phase to globally decrease with increasing length of the acoustic path. Furthermore, the algorithm investigates the reliability of possible candidates in D(iii) before choosing the next reference position.

## Results

Spherical and spectral phase unwrapping were applied to a set of dummy head (HEAD acoustics HSU III) HRTFs on a full-sphere grid with 1014 positions (left-ear filter set) measured in the anechoic chamber at IRCAM. The spherical coordinates used for appropriate depiction are the *lateral angle* $\alpha$ (from $0°$ on the interaural axis, ipsilateral side, to $180°$ on the interaural axis, contralateral side) and the *polar plane angle* $\beta$ (from $0°$/front via the upper hemisphere in positive direction to $180°$/back and via the lower hemisphere in negative direction to $-180°$/back). In order to evaluate the performance of

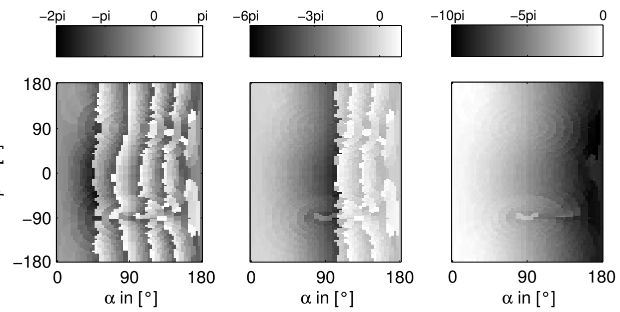


**Figure 1:** Progression of the spherical phase unwrapping algorithm for 7kHz. Results after iterations 100, 400 and 685 (left to right).

the proposed algorithm, a *phase discontinuity measure* is introduced. It is calculated by counting the absolute unwrapped phase differences greater than $\pi$ between each position and its neighbors and dividing the result by the number of neighbors. The result is then multiplied by 100 and hence delivers the amount of phase discontinuity for each position in percent.
In comparison to spectral phase unwrapping, fig. 2 shows a globally reduced number of discontinuities for spherical phase unwrapping and its tendency to shift them to the contralateral side. Fig. 3 depicts the frequency dependence of the phase discontinuity measure. Obviously, spherical phase unwrapping eliminates the low-frequency discontinuities contained in the spectrally unwrapped phase. The discontinuity measure increases with frequency for both unwrapping approaches, but is less steep

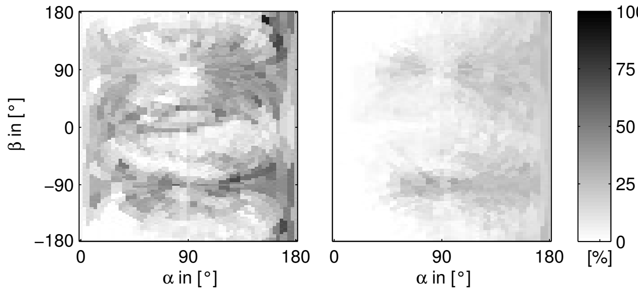


**Figure 2:** Spatial distribution of the phase discontinuity measure produced by spectral (left) and spherical phase unwrapping (right) - mean over frequency ($0 - 22.05$kHz).

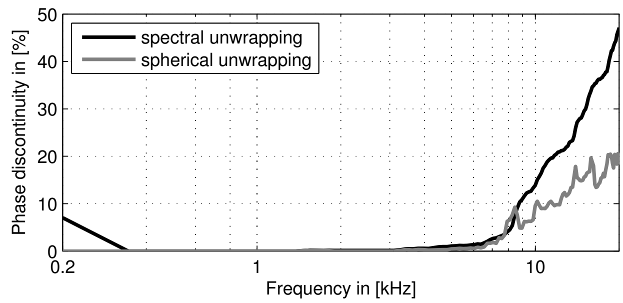


**Figure 3:** Spectral distribution of the phase discontinuity measure - mean over positions.

for the spherical approach. This is because the phase is spatially undersampled for small wavelengths and the spherical algorithm enforces aliasing of the true absolute phase to a contour which is continuous on the discretization grid.

## Conclusion and Outlook

In this contribution we proposed to apply phase unwrapping on the sphere to improve phase interpolation of HRTFs. The algorithm enforces spatial phase continuity based on a roughly rotationally symmetric phase distribution of HRTFs with respect to the interaural axis, which decreases towards the contralateral side. It is easy to adapt for directivity functions of any kind that usually provide similar a priori knowledge.
As the approach is based on a spatial phase sampling constraint, it might fail for high frequencies due to spatial undersampling of the phase. This can be avoided using a physical model for an estimation of the absolute phase that facilitates phase aliasing suppression, e.g. [4].

## References


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