EXPERIMENTAL AND THEORETICAL STUDY OF THE VIBRATION OF STRINGS IN THE HIGH REGISTER OF THE PIANO THE EFFECT OF THE "DUPLEX SCALE".

PACS 43.75.Mn

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ABSTRACT

The quality of sound in terms of duration and level is not satisfactory in the high register of the piano, and unlike the other registers, dampers are not necessary. This problem is related, mainly, to the coupling between the strings, the bridge, and the soundboard. To enhance the tonal quality of this register, some piano makers add extra sound produced by the oscillation of the short rear section of strings, properly tuned (octave, fifth or unison) in line with the vibrating section and driven by the latter through the bridge (this is called "duplex scale"). To analyze the effect of the "duplex scale", experimental and theoretical studies were carried out on a real piano and on an individual sample of a Grand Piano built by the Tallinn Piano Factory for A. Stulov of the Institute of Cybernetics, specially prepared for experimental measurements. Inspired by the model of nonconservately coupled oscillators of G. Weinreich (J. Acoust. Soc. Am. 62, N°6, 1977), extended to a third oscillator, we investigate the energy transmission between the string coupled through the bridge and analyze the effect of the "duplex scale" in the high register. Results will be presented and discussed. [Support from Egide-Parrot program, project N°82/239584, is acknowledged].

INTRODUCTION

Quality of sound

In concert pianos, three strings are required in a large range of notes to increase the energy transmitted to the soundboard in order to homogenize the acoustic power radiated from each piano note, from the lowest to the highest. In the high register, sound decreases very quickly after the hammer's strike. This is due to the intrinsic damping of the string and to the energy transferred to the soundboard. Piano manufacturers try to find a compromise between duration (sustain) and acoustic power radiated (loudness) but the aim is to increase the performances in both terms, duration and also acoustic power radiated.

Phenomenon explanation

Duration and radiated acoustic power are linked, amongst other things and for a same excitation, to the string intrinsic damping, the mistuning between the strings and the resulting coupling, the bridge impedance and the soundboard. One can find here some of the parameters that leads to better or worse quality.

We focus on the string's coupling problem in high register where we can find several different mechanical architectures. Some piano makers have tried to enhance the piano sound quality by adding spectral components.For example, Duplex scale (Steinway) or Aliquot (Blüthner) are some patents used today to achieve this aim. On the other hand, Bösendorfer does not use any artefact in their pianos to improve the sound.

Duplex scales consist of using the dead parts of the strings and adjusting them in respect to the vibrating part (see figure 1). In most cases, they are tuned by the manufacturer (unison, octave, or fifth) and the position of the bridges (used to fix the length of the string and therefore the frequency) is never changed.

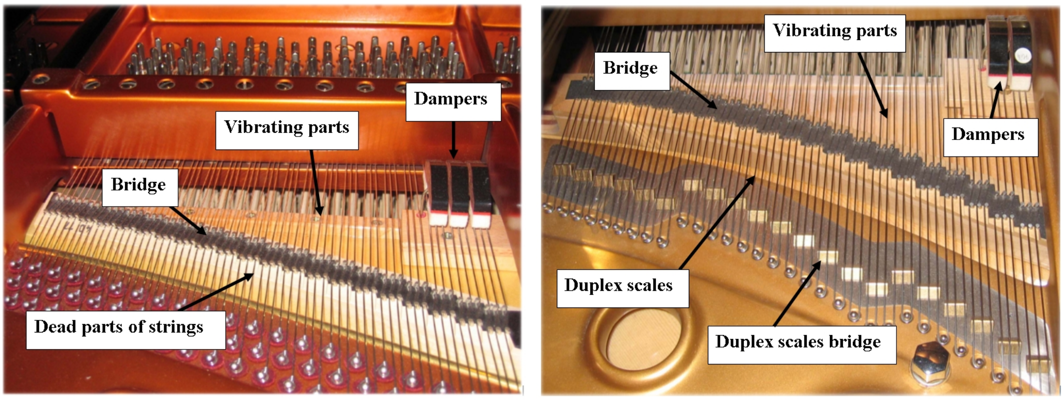

Figure 1: Piano high register architecture without and with Duplex scales

Previous studies

A few studies have been carried out around this subject. Kirk [4] or Marandas [1] have measured the mistuning of the strings in medium register. It was found that mistuning is precisely adjusted to control the sound decrease and make appear double decay pattern (prompt-sound and after-sound).
Weinreich [7] proposed a simple model to reproduce the behaviour of the bridge excited by the vertical component of a doublet vibration. This model is based on two mass-spring systems coupled by a bridge impedance. It is used to explain the phenomenon of double decay in the medium register.
Wogram [8] measured the impedance of a soundboard but, due to the measurement conditions, results weren't good at high frequencies. Giordano [3] and Suzuki [6] have measured the impedance of a soundboard at low frequencies most for modal analysis. Giordano [2] also proposed a simple model of the soundboard.

Aim of this study

Few notes of the piano use doublets of strings and those are implemented only in the low medium register. We decide to implement in the Weinreich model a third mass-spring system to be closer to reality in high register and also to simulate the influence of a passive string on the global behaviour of the doublet.
We try to find a double decrease behavior in medium-high register with this new triplet model, adjusting the mistuning values of each mass-spring system's resonance frequency with mistuning values from a previous study [1]. For our high register interest, in collaboration with piano tuners for the adjustment

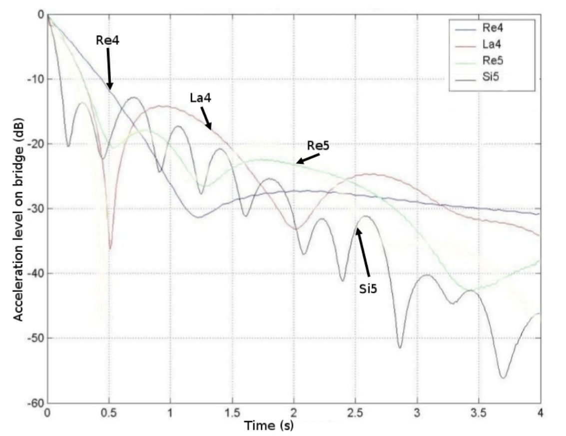

Figure 2: Time decrease profiles for some notes of the high register

in the unison of the triplets, we also find the information of mistuning between each string of high register triplets and this makes it possible to simulate the behavior of the triplet with measured values. Profiles of time decrease measurements are presented in figure 2. We easily see a double decrease for D^4 among the others irregular decreases (the piano used for measurements was not recently tuned). However, double decay phenomenon can be present only in a certain range of notes in the medium-high register. In the high register, decrease is to fast and tuners adjust the triplet only looking for the best compromise.

PHYSICAL MODELLING

Model with two coupled mass-spring oscillators

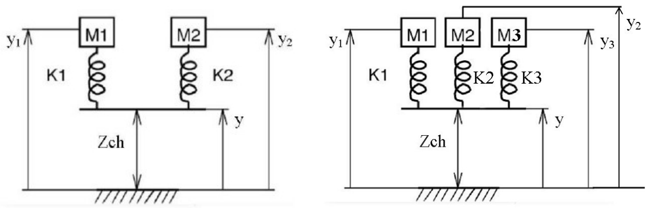

Figure 3: Coupled mass-spring systems

A simplified model implemented by Thomas [5] is pointed out hereafter (see figure 3, left-end diagram). It takes, as a starting point, the Weinreich model [7] which study coupled piano strings taking into account only the fundamental mode of string vibration (mass-spring system without damping). The model uses bridge impedance defined as constant around the fundamental pulsation of the string.
Figure 4 (upper diagram) shows the behaviour of the doublet model function of mistuning. While the first mass-spring system has a fixed pulsation (red curves), pulsation of the second system evolves from $-3cents$ to $+3cents$ (blue curves). This highlights the coupling phenomenon (solid line) appearing in a certain range [-2cents;+2cents] compared to strings without coupling (dashed lines). We will discuss about the triplet behaviour later.

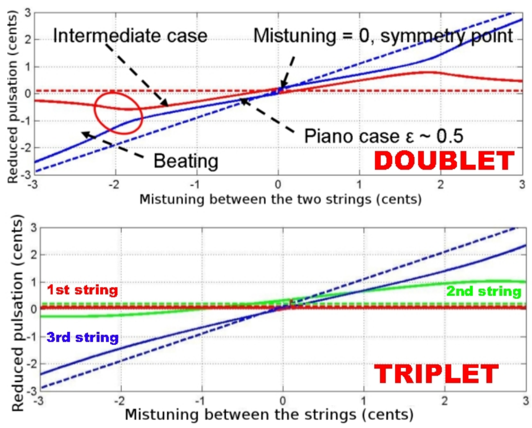

Figure 4: Behaviour of the two models, doublet and triplet, in function of mistuning

Model with three coupled mass-spring oscillators

To solve the problem described in figure 3 (right-end diagram), we solve the system 2 where bridge impedance is $Z_{br} = Force/Velocity$. The force of the three springs K_1, K_2, and K_3 on bridge is written as

$$F_{str/br} = K_1(y_1 - y) + K_2(y_2 - y) + K_3(y_3 - y) \tag{1}$$

If we suppose solutions in terms of $y = pe^{\lambda t}$, with $\lambda = -\alpha + i\omega$, the velocity is $V = \lambda y$.
The hypothesis that absorbtion rate α is negligible in front of the pulsation ($0 < \alpha < 10$, classical values) is done. Consequently we obtain the following system:

$$\begin{cases} M_1\ddot{y_1} + K_1 y_1 - K_1 y = 0 \\ M_2\ddot{y_2} + K_2 y_2 - K_2 y = 0 \\ M_3\ddot{y_3} + K_3 y_3 - K_3 y = 0 \\ i\omega_0 y Z_{ch} = K_1(y_1 - y) + K_2(y_2 - y) + K_3(y_3 - y) \end{cases} \tag{2}$$

Eliminating y in the first three equations, we find the characteristical equations of a system with three degrees of freedom. This system can also be written in the following matrix form: $\mathbf{M}\ddot{\vec{Y}} + \mathbf{K}\vec{Y} = 0$ with $\vec{Y} = (y_1\ y_2\ y_3)^t$ and

$$\mathbf{M} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} \quad , \quad \mathbf{K} = \frac{1}{K_1 + K_2 + K_3 + i\omega_0 Z_{ch}} \begin{pmatrix} A1 & -B & -C \\ -B & A2 & -D \\ -C & -D & A3 \end{pmatrix}$$

with:

$$\begin{array}{ll} A1 = K_1(K_2 + K_3 + i\omega_0 Z_{ch}) & B = K_1 K_2 \\ A2 = K_2(K_1 + K_3 + i\omega_0 Z_{ch}) & C = K_1 K_3 \\ A3 = K_3(K_1 + K_2 + i\omega_0 Z_{ch}) & D = K_2 K_3 \end{array}$$

We have to solve the equation $det(\lambda^2 \mathbf{Id} + \mathbf{M}^{-1}\mathbf{K}) = 0$, that means finding the proper values l_i of the matrix $\mathbf{M}^{-1}\mathbf{K}$.
We obtain three complex values λ^2, with $\lambda_i^2 = -l_i$.
As we need a stable system, and therefore damped vibrations, the three solutions are $\lambda_i = -\alpha_i + i\omega_i$, with $\alpha_i > 0$ (because $e^{\lambda_i} = e^{Re(-\lambda_i)} + e^{i.Im(\lambda_i)}$).
Now, we have $(\vec{P_i}, \lambda_i)$, with $\vec{P_i}$ the proper vectors and so, the complex general solution is:

$$\vec{Y} = \sum_{i=1}^{3} A_i \vec{P_i} e^{-\alpha_i t} e^{i\omega_i t} \qquad (3)$$

As we can see in figure 4 (lower diagram) behaviour of the triplet model no longer shows any symmetry because of the new string (tuned near from the first string, $\epsilon = 0.1cent$). It is not also possible to clearly locate the coupling areas. We will see later that time behaviour of this model is the interesting part, much more than for the doublet.

EXPERIMENTATION

For our study we need to know mean mistuning values for some triplets of strings. A microphone is placed close to the strings of a piano's high register. We adjust some triplets with the help of a tuner. Then, blocking two strings with special felt, we can measure only the sound radiated by only one string. With this method, we measure separately each string and then the global triplet sound radiated. Using the zero-cross method, we determine instantaneous frequency of each string. This technique is used by Marandas [1] and enabled to precisely measure the mistuning of a triplet strings. Mean mistuning values on four high register triplets and one medium-high register, previously adjusted by a tuner, are presented in table 1. Even if we didn't measured mistunings for the same notes, values follow the same logic, always two close pulsations and the third, more distant.

	$E_\flat^4$	$G_\sharp^6$	C^7	$A_\flat^7$	C^8
Left string	0	1.8	0	0	0
Middle string	0.1	0.1	0.1	1	0.4
Right string	0.65	0	0.4	2.2	1.6

Table 1: Mean values of mistuning measured on a grand piano

NUMERICAL TIME BEHAVIOUR SIMULATION

Comparison between the two models

We compare the behaviour of the two models in the medium-high register, using measured mistuning values of $E_\flat^4$ from Marandas [1]. Bridge impedance is set with values measured on medium-high register, $Z_{ch} = 20kg.s^{-1}$, used in a previous study [5]. For the doublet model (see figure 5, left-upper figure), the phase between the strings evolves from one "in phase" system at the hammer's impact to a sort of "out of phase" stability, coupled mass-spring systems exchanging their energy.
For the triplet model (see figure 5, left-lower figure), the phase evolves as for the doublet for string 2 and string 1 but the influence of string 3 exists. This system of rotating phases has an influence on the profile of time decay of each string. In figure 5 (right-end figure), one can find the phenomenon

of double decay. The role of the third string does not change completely the behaviour. Changes are smoother than for the doublet but decay profiles are comparable. In medium-high register, doublet and triplet models show good agreement in the reproduction of time double decay.

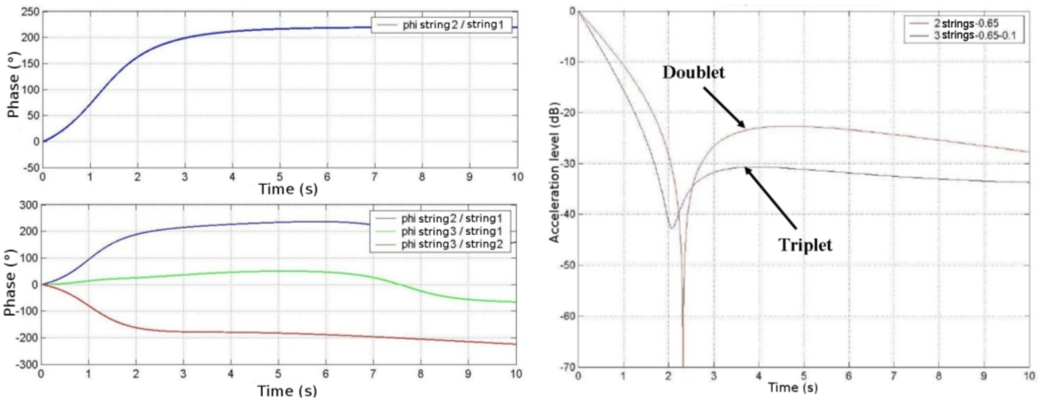

Figure 5: Behaviour of a the two models, phase between strings and global mouvement of the bridge, for $E_\flat^4$ simulation

Study of the triplet model for high register, simulation with C^8 parameters

We are now interested in the effect of mistuning in a high register triplet. We apply different mistunings with C^8 parameters (length of string, diameter, Young modulus, etc...) on the triplet model. Bridge impedance is now set with measured value on high register, $Z_{ch} = (40 + 40i)kg.s^{-1}$. The imaginary part of the impedance is not zero (as for medium-high register simulation) as soundboard acts as a non-conservative mass-spring system and does reinject energy into the strings. This value of the bridge

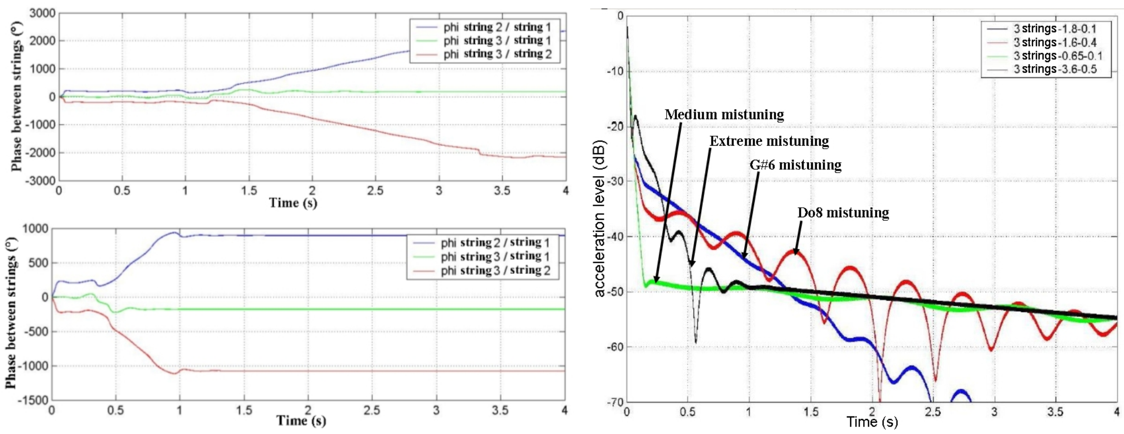

Figure 6: Time phase profiles of C^8 with different mistuning values and time decrease profiles function of mistuning for C^8

impedance is the result of our measurements with impact hammer. In figure 6, we adjust $G_\sharp^6$ mistuning values (left-upper figure) and C^8 values (left-lower figure) and look at the phase evolution with time.
While medium-high register mistuning values help to install double decay phenomenon, in high register mistuning helps installing a quick phase rotation system that aims to keep energy in the strings and modify the decrease profile during the first seconds.
For $G_\sharp^6$ mistuning values, phase rotation phenomenon starts at approximatively $1.2s$ and continues until $3.4s$.
For C^8 mistuning values, phase rotation phenomenon appears sooner. On high register, energy is quickly transfered to the soundboard or damped by the strings and we must try to install phase rotation as soon as possible, when most of the energy can be conserved into the system.
We perform a simulation with four mistuning values (see legends on figure 6). For each one of these mistunings, global behaviour is really different but it's hard to conclude on an ideal mistuning. Values

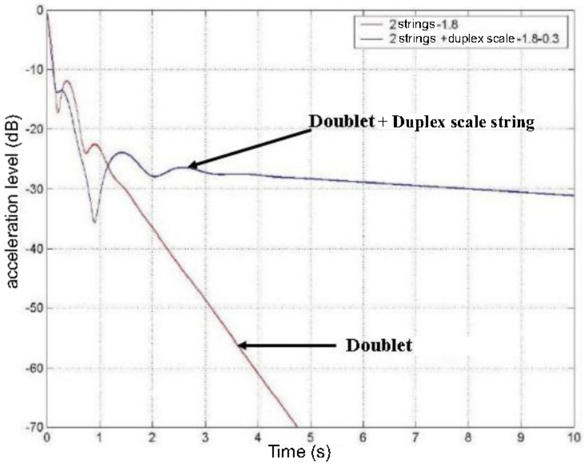

Figure 7: Time decrease profiles for a doublet with and without a single duplex scale string for $G^6_\sharp$

of mistuning depend on many parameters. For example, changing the bridge impedance value will have a strong influence on mistuning values needed to observe the same global behaviour of the triplet.

Study of the doublet model with a third passive mass-spring system (Duplex scale)

Now, we simulate the behaviour of a doublet of $G^6_\sharp$ strings with, and without, a passive third coupled string, i.e. non struck by the piano hammer. We use triplet model in which we set to zero the original velocity of the third string. We obtain therefore the behaviour of a doublet of strings with a "duplex scale" string, tuned to the unison (approximatively same fundamental frequency). Figure 7 shows the influence of the duplex scale string. The effect is that third string allows phase rotation phenomenon and helps to keep energy in the coupled strings system. Because of our model (mono frequency excitation of the bridge) we only present simulation with duplex scale string tuned to unison.

CONCLUSION

First we present a triplet model using mass-spring systems coupled to a bridge impedance. Compared to the previous model of a doublet in the medium-high register, behaviour is comparable in medium register. Then we use triplet model to better understand the behaviour in high register. We show that mistuning between each string induces a system of rotating phases which leads to keep energy into the coupled strings. The aim of the tuner is to adjust the mistuning in order to install quickly this rotating phase phenomenon to control the decrease profile of the note.

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